INCORPORATING GRAVITY INTO TRACE DYNAMICS

THE INDUCED GRAVITATIONAL ACTION

TRACE DYNAMICS:

OPERATOR HAMILTONIAN $H[\{q_n\},\{p_n\}]$

TRACE HAMILTONIAN

$$\underset{\sim}{H} = \mathrm{Tr}\, H[\{q_n\},\{p_n\}]$$

FOR ANY TRACE FUNCTIONAL $\underset{\sim}{A}[\{q_n\},\{p_n\}]$

DEF:

$$\delta A = \mathrm{Tr} \sum_n \left[\frac{\delta \underset{\sim}{A}}{\delta q_n} \delta q_n + \frac{\delta \underset{\sim}{A}}{\delta p_n} \delta p_n \right]$$

FROM $\underset{\sim}{H}$, GET A SYMPLECTIC DYNAMICS ON OPERATOR PHASE SPACE:

$$\frac{\delta \underset{\sim}{H}}{\delta q_n} = -\dot{p}_n \qquad \frac{\delta \underset{\sim}{H}}{\delta p_n} = \varepsilon_n \dot{q}_n$$

$\varepsilon_n = 1 \ (-1)$ BOSONIC (FERMIONIC) n

$\underset{\sim}{H}$ A CONSTANT OF MOTION

GLOBAL UNITARY INVARIANCE

$\Rightarrow$ CONSERVED NOETHER CHARGE

$$\tilde{C} = \sum_{n, B} [q_n, p_n] - \sum_{n, F} \{q_n, p_n\}$$

$d\mu$ = INTEGRATION MEASURE ON OPERATOR PHASE SPACE IS CONSERVED

SO CAN DO STATISTICAL MECHANICS

CANONICAL ENSEMBLE

$$d\mu\,\rho = \frac{d\mu\, e^{-\mathrm{Tr}(\tilde{\lambda}\tilde{C}) - \tau \underset{\sim}{H}}}{\int d\mu\, e^{-\mathrm{Tr}(\tilde{\lambda}\tilde{C}) - \tau \underset{\sim}{H}}}$$

AVERAGES OVER ρ LOOK LIKE QUANTUM MECHANICS WITH $\langle \tilde{C} \rangle_{\rho} = i_{eff}\, \hbar$

$$\dot{x}_{r\,eff} = \frac{i_{eff}}{\hbar}\left[H_{eff}, x_{r\,eff}\right] \qquad \left[q_{\mu\,eff}, p_{\nu\,eff}\right] = i_{eff}\, \hbar\, \delta_{\mu\nu}$$

- QUANTUM FIELD THEORY IS THERMODYNAMICS OF TRACE DYNAMICS

- FLUCTUATION CORRECTIONS (BROWNIAN MOTION) GIVE REDUCTION POSTULATE

INCLUDING GRAVITATION

ARGUMENTS FOR METRIC $g_{\mu\nu}$ BEING A C-NUMBER, NOT AN OPERATOR:

INVARIANT VOLUME

$$dV = d^4x \, \left({}^{(4)}g\right)^{1/2} \qquad {}^{(4)}g \equiv -\det g_{\mu\nu}$$

COORDINATE TRANSFORMATION: $x_\mu \to x_\mu(x')$

NEED PRODUCT PROPERTY OF DETERMINANT TO GET

$$({}^{(4)}g)^{1/2} \to ({}^{(4)}g')^{1/2} |J|$$

$$|J|\, d^4x = d^4x' \quad \Rightarrow \quad dV = dV'$$

PRODUCT PROPERTY FAILS IF $g_{\mu\nu}$ OPERATOR:

$$\det(O\ C) \neq \det O \det C$$

OPERATOR MATRIX

C-NO MATRIX

SUPERSYMMETRY

- TRACE DYNAMICS EXTENSIONS OF RIGID SUPERSYMMETRY THEORIES NEED C-NUMBER $g_{\mu\nu}$

- SUPERGRAVITY – NO TRACE DYNAMICS
  EXTENSION WITH OPERATOR $g_{\mu\nu}$, ψ_ν

SHOULD GRAVITY BE QUANTIZED?

- DYSON ARGUMENT: NO BOHR-ROSENFELD ANALOG, HARD TO FORMULATE EXPERIMENT TO DETECT SINGLE GRAVITON

- PAGE + GEILKER, EPPLEY + HANNAH
  PROBLEMS WITH $G_{\mu\nu} = -8\pi G \langle\psi| T_{\mu\nu} |\psi\rangle$
  "SEMI-CLASSICAL" GRAVITY
  WHEN MEASUREMENTS ARE MADE

MORE COMPLICATED FORMS OF CLASSICAL GRAVITY THEORIES ARE POSSIBLE

CLASSICAL GRAVITY IN TRACE DYNAMICS

$$\underset{\sim}{S}_m[g] = \int d^4x \, ({}^{(4)}g)^{1/2} \, \mathrm{Tr}\, \mathcal{L}(x; g)$$

$g_{\mu\nu}$ USED TO FORM COVARIANT DERIVATIVES

$$\underset{\sim}{S}_{TOT} = \underset{\sim}{S}_m[g] + \underset{\sim}{S}_g$$

$$\underset{\sim}{S}_g = \frac{\mathrm{Tr}(1)}{16\pi G} \int d^4x \, ({}^{(4)}g)^{1/2} \, R$$

DIVIDE OUT $\mathrm{Tr}(1)$:

$$S_g = \underset{\sim}{S}_g / \mathrm{Tr}(1)$$

$$S_m = \underset{\sim}{S}_m / \mathrm{Tr}(1)$$

GRAVITATIONAL FIELD EQUATIONS

$$G^{\mu\nu} + 8\pi G\, T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \frac{\underset{\sim}{T}^{\mu\nu}}{\mathrm{Tr}(1)} \qquad \underset{\sim}{T}^{\mu\nu} = \text{TRACE STRESS-ENERGY TENSOR}$$

$$\nabla_\mu \underset{\sim}{T}^{\mu\nu} = 0 \Rightarrow \text{CONSISTENCY WITH BIANCHI } \nabla_\mu G^{\mu\nu} = 0$$

IF TRACE DYNAMICS IMPLIES BOTH QUANTUM THEORY <u>AND</u> STATE VECTOR REDUCTION, THEN HAVE A CONSISTENT FRAMEWORK FOR CLASSICAL GRAVITY: BIANCHI MAINTAINED THROUGH MEASUREMENTS

MATTER-INDUCED EFFECTIVE ACTION FOR $g_{\mu\nu}$

AVERAGED PRE-QUANTUM MATTER MOTIONS CAN INFLUENCE GRAVITATIONAL DYNAMICS

$$S_{g;\,\mathrm{induced}} \equiv \int d^4x \, \left({}^{(4)}g\right)^{1/2} \frac{\mathrm{Tr}\,\langle \mathcal{L}(x)\rangle_{AV}}{\mathrm{Tr}\,(1)}$$

$$\langle \mathcal{L}(x)\rangle_{AV} \equiv \int d\mu\, \rho\, \mathcal{L}(x)$$

CAN SHOW ρ IS TIME-INDEPENDENT:

$$\tilde{C} = \int d^3x \left({}^{(4)}g\right)^{1/2} \tilde{C}^0 \qquad \nabla_\mu \tilde{C}^\mu = 0$$

$$\underset{\sim}{H} = \int d^3x \left({}^{(4)}g\right)^{1/2} \left[\underset{\sim}{T}_0{}^0(t,\vec{x}) + \mathrm{Tr}(1)\, t_0^0(t,\vec{x}) \right]$$

(EINSTEIN-DIRAC PSEUDOTENSOR

$$\partial_\nu \left[\left({}^{(4)}g\right)^{1/2} \left(\underset{\sim}{T}_\mu{}^\nu + \mathrm{Tr}(1)\, t_\mu^\nu \right) \right] = 0$$

CONSTRAINTS ON FORM OF EFFECTIVE ACTION

$\tilde{C}$ A LORENTZ SCALAR

$\underset{\sim}{H}$ TIME COMPONENT OF 4-VECTOR

$\Rightarrow$ CANONICAL ENSEMBLE PICKS A PREFERRED FRAME $\Leftrightarrow$ CMB REST FRAME

ρ INVARIANT UNDER PURELY SPATIAL GENERAL COORDINATE TRANSFORMATIONS

$\Rightarrow$ EFFECTIVE ACTION ALSO INVARIANT

g_{00} = 3-SPACE SCALAR

g_{0i} = 3-SPACE COVARIANT VECTOR

${}^{(4)}g / {}^{(3)}g = g_{00} + g_{0i} D^i$ $\qquad$ D^i = 3-SPACE CONTRAVARIANT VECTOR

SO LEADING ORDER EFFECTIVE ACTION IN EXPANSION IN POWERS OF DERIVATIVES OF THE METRIC IS

$$\Delta S_g \equiv S_{g,\text{induced}} = \int d^4x \left({}^{(4)}g\right)^{1/2} A\left(g_{00},\ g_{0i}\, g_{0j}\, g^{ij},\ D^i D^j g_{ij},\ g_{0i} D^i\right)$$

$A(a, b, c, d)$ GENERAL FUNCTION OF a, b, c, d

<u>NOW USE WEYL SCALING INVARIANCE</u> (FORGER + RÖMER)

GLOBAL WEYL : METRIC

$$g_{\mu\nu}(x) \to \lambda^2 g_{\mu\nu}(x)$$

$$g^{\mu\nu}(x) \to \lambda^{-2} g^{\mu\nu}(x)$$

$$e^a_\mu(x) \to \lambda\, e^a_\mu(x)$$

$$e^\mu_a(x) \to \lambda^{-1} e^\mu_a(x)$$

GLOBAL WEYL: MATTER $q(x)$ FIELD

$p(x)$ CANONICAL MOMENTUM

$$q(x) \rightarrow \lambda^{-W_q} q(x) \qquad p(x) \rightarrow \lambda^{-W_p} p(x)$$

	IN n DIMENSIONS	4 DIMENSIONS
SCALAR q	$W_q = \frac{1}{2}(n-2)$ $\quad W_p = W_q + 2$	$W_q = 1$ $\quad W_p = 3$
DIRAC SPINOR q	$W_q = \frac{1}{2}(n-1)$ $\quad W_p = W_q + 1$	$W_q = \frac{3}{2}$ $\quad W_p = \frac{5}{2}$
YANG-MILLS q	$W_q = W_p = 0$	$W_q = W_p = 0$

MASSLESS SPIN-0 ACTION
MASSLESS DIRAC SPINOR ACTION
YANG-MILLS ACTION

} ALL GLOBALLY WEYL INVARIANT <u>OFF</u>-SHELL IN $n = 4$

$$\left.\begin{array}{l} ({}^{(4)}g)^{1/2}\ T_\lambda{}^\nu \\[2ex] ({}^{(4)}g)^{1/2}\ \tilde{C} \end{array}\right\}$$ GLOBALLY WEYL INVARIANT OFF-SHELL

$\Rightarrow$ CANONICAL ENSEMBLE $\dfrac{d\mu\, \rho}{\int d\mu\, \rho}$ IS WEYL INVARIANT

SO EFFECTIVE ACTION ΔS_g IS WEYL INVARIANT

$$\Delta S_g = \int d^4x\, ({}^{(4)}g)^{1/2}\, g_{00}^{-2}\, A\left(g_{0i}\, g_{0j}\, g^{ij}/g_{00},\ D^i D^j g_{ij}/g_{00},\ g_{0i} D^i/g_{00}\right)$$

- $S_{\text{cosmological}} \propto \int d^4x\, ({}^{(4)}g)^{1/2}$ EXCLUDED
- WHEN $g_{0i} = g^{0i} = D_i = 0$,

$$\boxed{\Delta S_g = A_0 \int d^4x\, ({}^{(4)}g)^{1/2}\, (g_{00})^{-2}}$$

RULES FOR USING FRAME-DEPENDENT ACTION

$$S_{TOTAL} = S_g + \Delta S_g + S_{pm}$$

PARTICULATE MATTER

FRAME DEPENDENT
ONLY SPATIAL COORDINATE INVARIANT

- $$G^{ij} + 8\pi G\left(\Delta T^{ij} + T^{ij}_{pm}\right) = 0$$

$$\delta \Delta S_g = -\frac{1}{2}\int d^4x\, ({}^{(4)}g)^{1/2}\, \Delta T^{ij}\, \delta g_{ij}$$

- G^{0i}, G^{00} EQUATIONS OBTAINED BY BIANCHI

$\Rightarrow$ CONSERVING EXTENSIONS

ΔT^{i0} $\quad \Delta T^{00}$ $\quad$ OF $\quad \Delta T^{ij}$

APPLICATION TO ROBERTSON-WALKER COSMOLOGY

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right]$$

$$g_{00} = 1 \quad \Rightarrow \quad \Delta S_g = A_0 \int d^4x \, \left({}^{(4)}g \right)^{1/2}$$

$$\Delta T^{ij} = - A_0 \, g^{ij}$$

CONSERVING EXTENSION $\Delta T^{\mu\nu} = - A_0 \, g^{\mu\nu}$

LOOKS LIKE A COSMOLOGICAL CONSTANT!

IF NO "BARE" COSMOLOGICAL CONSTANT, SO-CALLED "DARK ENERGY" IS THE ENERGY ASSOCIATED WITH MOTIONS OF PRE-QUANTUM MATTER FIELDS

$$G^{\mu\nu} + \Lambda g^{\mu\nu} + 8\pi G \, T^{\mu\nu}_{pm} = 0 \quad \Rightarrow \quad A_0 = -\frac{\Lambda}{8\pi G}$$

APPLICATION TO STATIC, SPHERICALLY SYMMETRIC METRIC

$$ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$g_{00} = B(r) \quad \Rightarrow \quad \Delta S_g = -\frac{\Lambda}{8\pi G}\int d^4x\,(-g)^{1/2}\, B(r)^{-2}$$

$$\delta g_{ij} \Rightarrow \quad \Delta T^{ij} = \frac{\Lambda}{8\pi G}\frac{g^{ij}}{B(r)^2} \qquad \Delta T_{ij} = \frac{\Lambda}{8\pi G}\frac{g_{ij}}{B(r)^2}$$

MODIFIED EINSTEIN EQUATIONS

$$G_{rr} - \frac{\Lambda\, A(r)}{B(r)^2} = 0$$

$$G_{\theta\theta} - \frac{\Lambda\, r^2}{B(r)^2} = 0$$

CONSERVING EXTENSION

BIANCHI IDENTITY

$$G'_{rr} - \frac{2A}{r^3} G_{\theta\theta} + \left(\frac{B'}{2B} + \frac{2}{r} - \frac{A'}{A}\right) G_{rr} + \frac{AB'}{2B^2} G_{tt} = 0$$

$\Rightarrow$ MODIFIED EQUATION FOR G_{tt} IS

$$G_{tt} - \frac{3\Lambda}{B} = 0$$

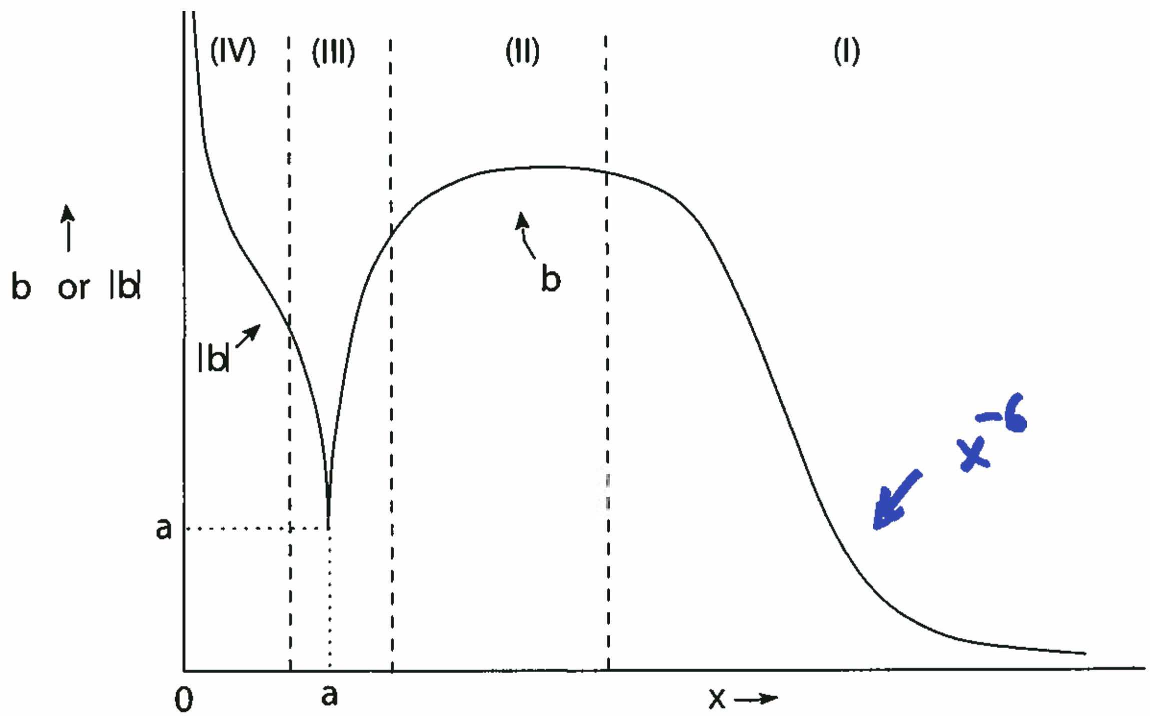
THIS SYSTEM OF EQUATIONS UNDER STUDY:

$$x = \Lambda^{1/2} r \qquad A(x) = \frac{B^2 + xBB'}{B^2 - x^2}$$

$$B'' + \frac{2}{x} B' + 2 \frac{(xB' + B)(xB' + 3B)}{B(B^2 - x^2)} = 0$$

QUALITATIVE BEHAVIOR

$b = B(x)$



- $R \equiv 0$

- $b(a)$ = SQUARE ROOT BRANCH POINT
  (COORDINATE SINGULARITY)

- LARGE x CURVATURE SINGULARITY - STATIC ARTIFACT?

SOME REFERENCES

EFFECTIVE ACTION

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WEYL SCALING

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