Entropic Dynamics: an Inference Approach to Time and Quantum Theory

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Question:

Do the laws of Physics reflect Laws of Nature? Or...

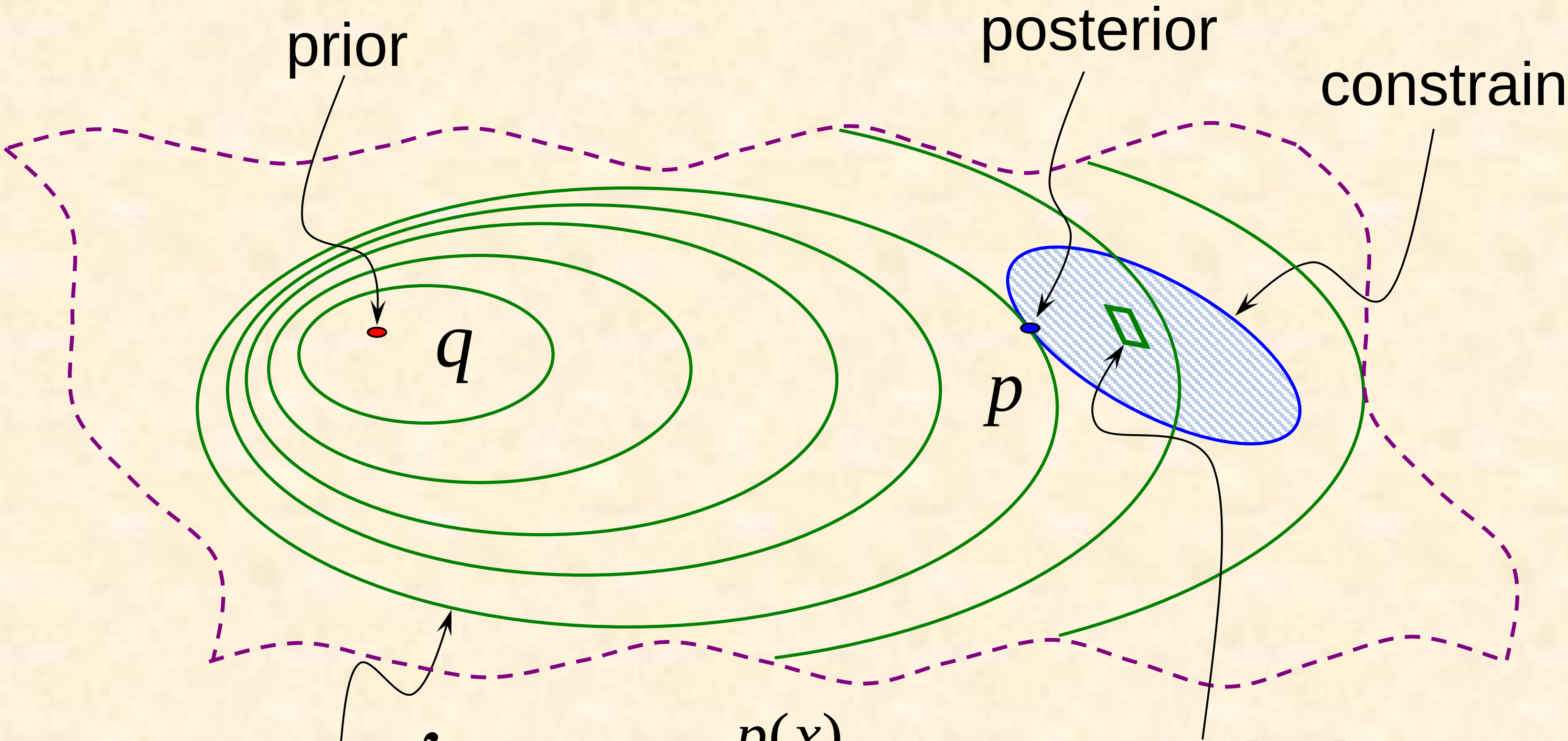
Are they rules for processing information about Nature?

Our objective:

To derive Quantum Theory as Entropic Dynamics

and discuss some implications for the theory of time, quantum measurement, Hilbert spaces, etc.

Entropic Inference:

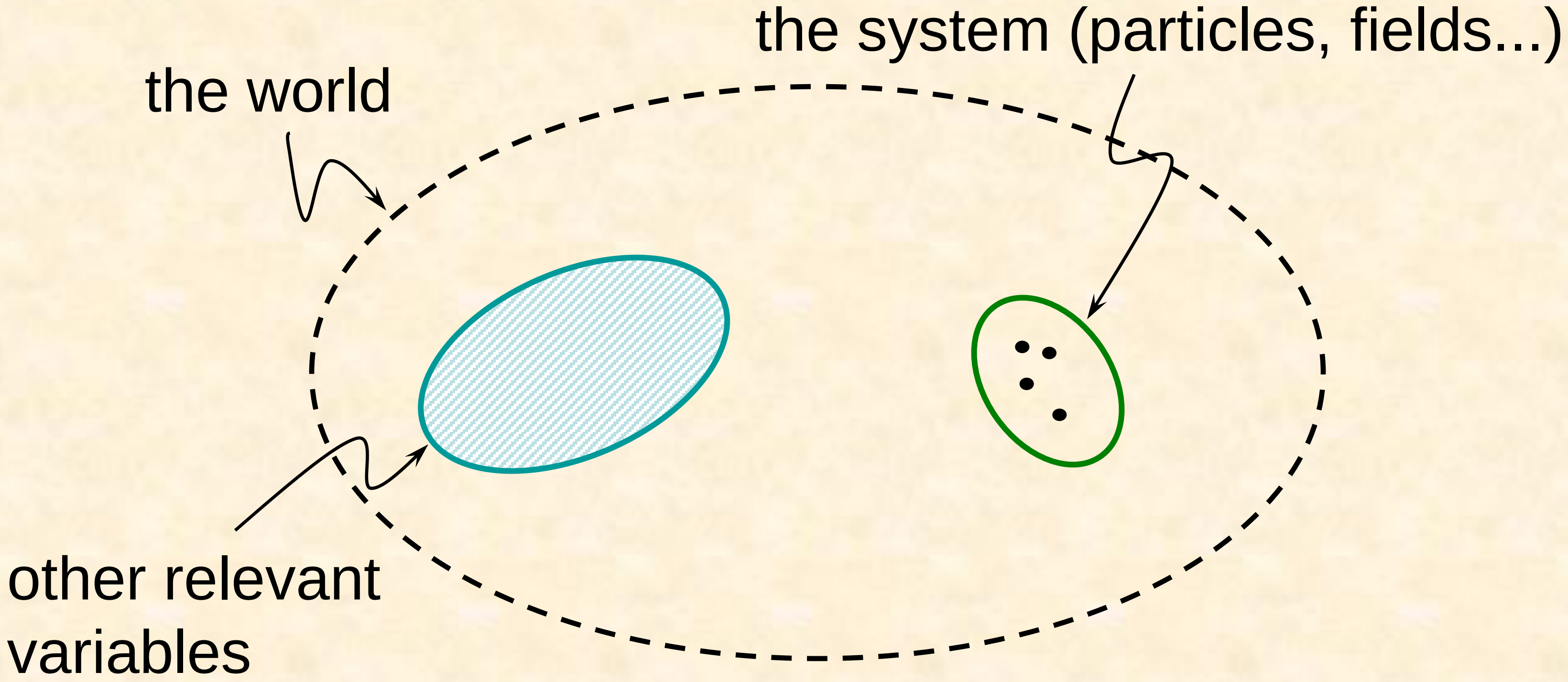


$$S[p,q] = -\int dx\, p(x) \log \frac{p(x)}{q(x)} \qquad \Pr(dV) \propto e^{S[p,q]} dV$$

Maximize $S[p,q]$ subject to the appropriate constraints.

(MaxEnt, Bayes' rule and Large Deviations are special cases.)

The subject matter



The subject matter

The goal is to predict the positions of particles, x.

Particles have definite but unknown positions. $\Rightarrow \quad \rho(x)$

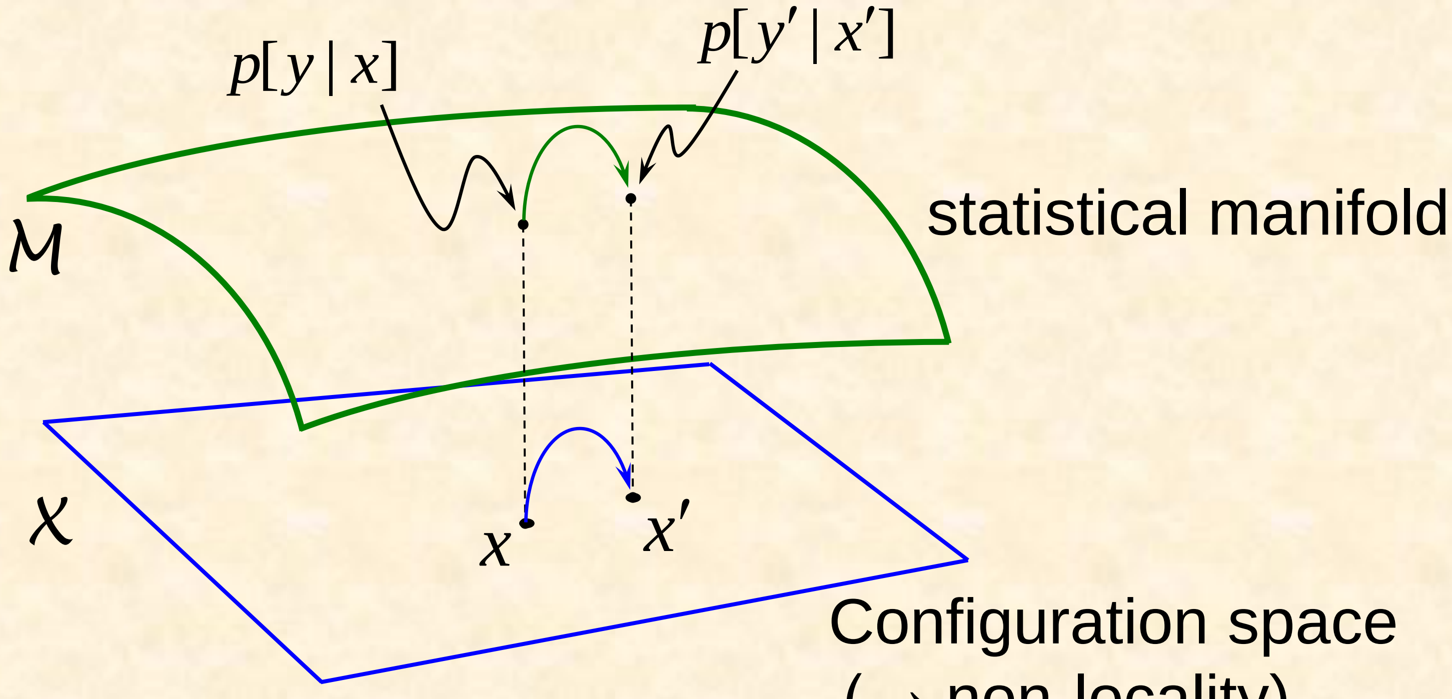
There exist other variables y with probability $p(y|x)$

and entropy

$$S(x) = -\int dy\, p(y\,|\,x) \log \frac{p(y\,|\,x)}{q(y)} \quad \Rightarrow \quad \phi(x)$$

(y variables are **not** hidden variables.)

Dynamics: Change happens.



Entropic Dynamics

Maximize the joint entropy

$$S_J[P,Q] = -\int dx'dy'\, P(x',y'|x) \log \frac{P(x',y'|x)}{Q(x',y'|x)}$$

uniform

$$P(x'|x)\, p(y'|x')$$

$\in \mathcal{M}$

Short steps: $\langle \Delta\ell^2 \rangle = \langle \gamma_{ab} \Delta x^a \Delta x^b \rangle = \Delta\lambda^2$

The result:

$$P(x'|x) = \frac{1}{\zeta} \exp[S(x') - \frac{1}{2}\alpha(x)\Delta\ell^2]$$

where $$S(x') = -\int dy'\, p(y'|x') \log \frac{p(y'|x')}{q(y')}$$

Displacement: $$\Delta x = \Delta\bar{x} + \Delta w$$

Expected drift : $$\Delta\bar{x}^a = \frac{1}{\alpha(x)} \nabla^a S(x) \quad \leftarrow O(\alpha^{-1})$$

!!!

Fluctuations: $$\langle \Delta w^a \Delta w^b \rangle = \frac{1}{\alpha(x)} \gamma^{ab} \quad \leftarrow O(\alpha^{-1/2})$$

Entropic Time

The foundation of any notion of time is dynamics.

Time is introduced to keep track of the accumulation of many small changes.

$$P(x') = \int dx\, P(x', x) = \int dx\, P(x' \mid x) P(x)$$

(1) Introduce the notion of an instant

$$\rho(x', t') = \int dx\, P(x' \mid x) \rho(x, t)$$

(2) Instants are ordered: the Arrow of Entropic Time

(3) Duration: the interval between instants

For large α the dynamics is all in the fluctuations:

$$\langle \Delta w^a \Delta w^b \rangle = \frac{1}{\alpha}\gamma^{ab} = \frac{\hbar}{m}\Delta t \delta^{ab}$$

Define duration so that motion looks simple: $\alpha = \dfrac{\tau}{\Delta t}$

Entropic dynamics:

$$\rho(x',t') = \int dx\, P(x' \mid x)\rho(x,t)$$

Fokker-Planck equation:

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v})$$

Fokker-Planck equation: $$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v})$$

$$\vec{v} = \frac{\hbar}{m} \vec{\nabla} \phi \qquad \phi(x,t) = S(x) - \log \rho^{1/2}(x,t)$$

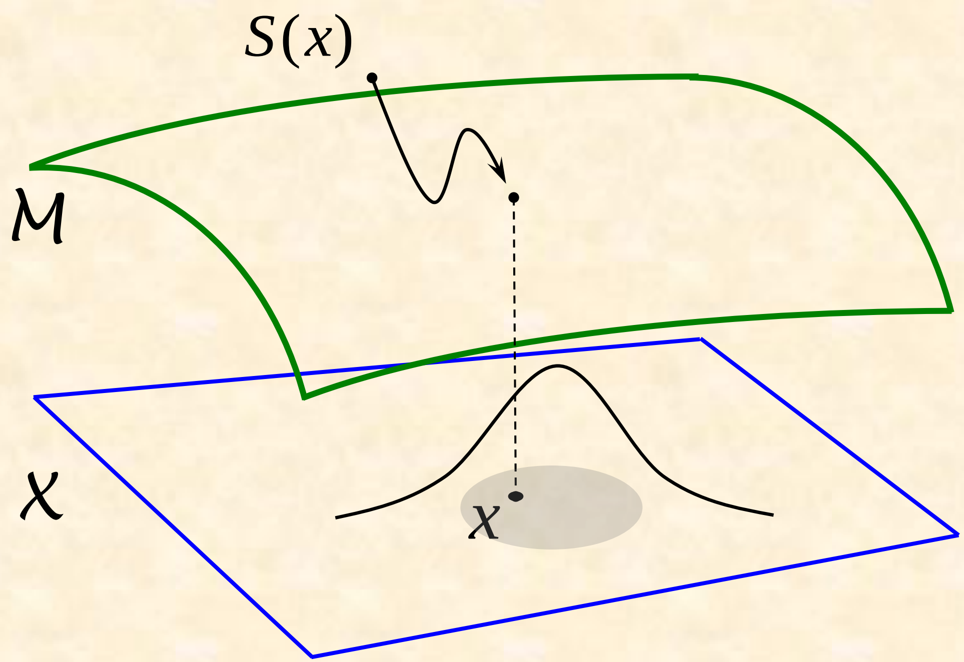
$$\vec{v} = \vec{b} + \vec{u}$$

drift velocity: $$\vec{b} = \frac{\hbar}{m} \vec{\nabla} S$$

osmotic velocity: $$\vec{u} = -\frac{\hbar}{m} \vec{\nabla} \log \rho^{1/2}$$

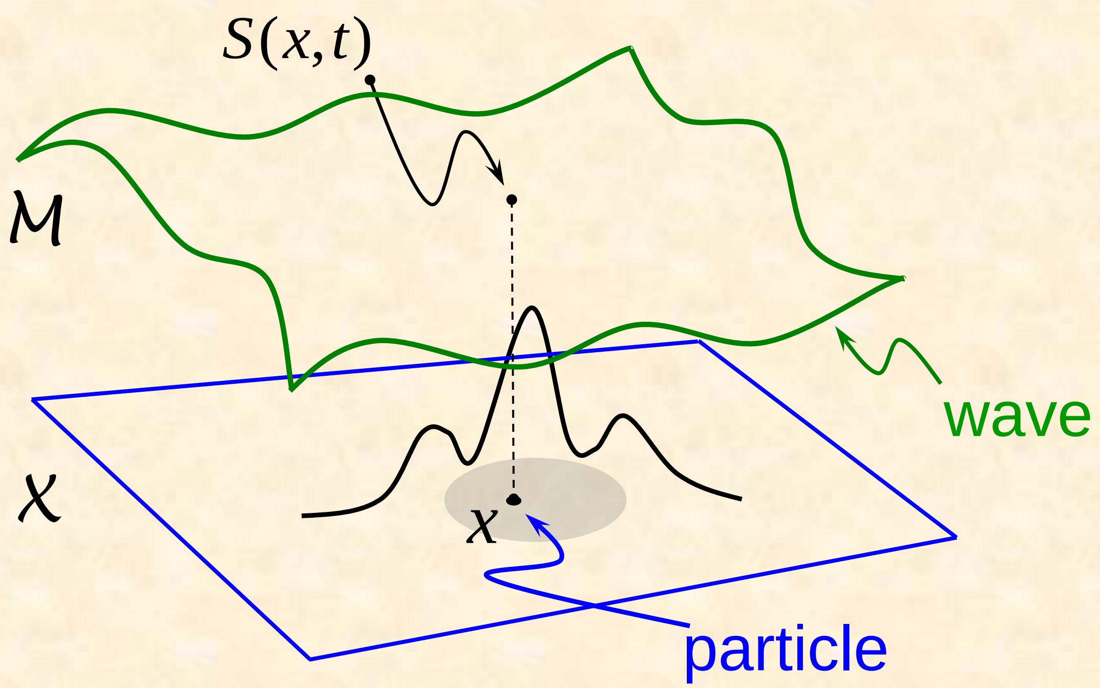
diffusion current: $$\rho \vec{u} = -\frac{\hbar}{2m} \vec{\nabla} \rho$$

Entropic dynamics:



Problem: this is just standard diffusion, not QM!

The solution: allow $\mathcal{M}$ to be dynamic.



Non-dissipative diffusion ⇨ Quantum Mechanics

Manifold dynamics?

Impose "energy" conservation

$$E[\rho, S] = \int d^3x\, \rho \left[\frac{1}{2} m v^2 + \frac{1}{2} m u^2 + V(x) \right] \qquad \text{!!!}$$

Quantum Hamilton-Jacobi equation:

$$\hbar \dot{\phi} + \frac{\hbar^2}{2m} (\nabla \phi)^2 + V - \frac{\hbar^2}{2m} \frac{\nabla^2 \rho^{1/2}}{\rho^{1/2}} = 0$$

The result: two coupled equations

1) Fokker-Planck/diffusion equation

$$\frac{\partial \rho}{\partial t} = -\frac{\hbar}{m} \nabla \cdot \left(\rho \nabla \phi \right)$$

2) Quantum Hamilton-Jacobi equation

$$\hbar \dot{\phi} + \frac{\hbar^2}{2m} (\nabla \phi)^2 + V - \frac{\hbar^2}{2m} \frac{\nabla^2 \rho^{1/2}}{\rho^{1/2}} = 0$$

Combine ρ and ϕ into $\Psi = \rho^{1/2} e^{i\phi}$

to get Quantum Mechanics, $i\hbar\dot{\Psi} = -\dfrac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$

Complex numbers, linearity, and Hilbert spaces are convenient but not fundamental.

Measurement of position in ED

Particles have definite but unknown positions.

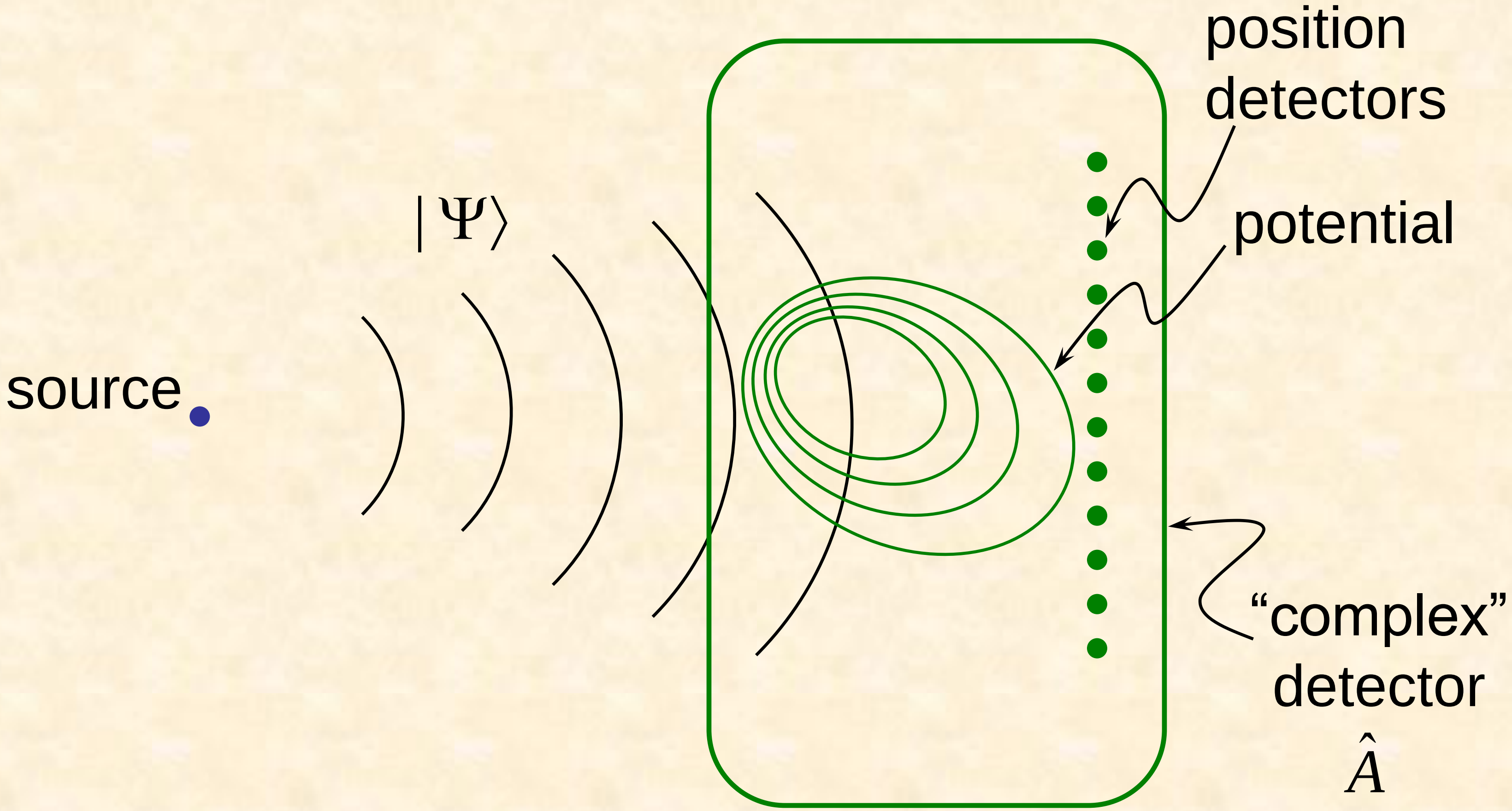
Born rule: $$\rho(x)dx = \left|\Psi(x)\right|^2 dx = \left|\langle x \,|\, \Psi \rangle\right|^2 dx$$

Measurement of position is “classical”:

a mere ascertaining of pre-existing values,

a mere amplification from micro to macro scales.

What about other “observables”?



$$|\Psi\rangle \rightarrow \hat{U}_A |\Psi\rangle \qquad p_i = \left| \underbrace{\langle x_i | \hat{U}_A}_{\langle a_i |} | \Psi\rangle \right|^2$$

Other "observables" and the Born rule:

If $|a_i\rangle \quad \rightarrow \quad \hat{U}_A |a_i\rangle = |x_i\rangle$

then $|\Psi\rangle = \sum c_i |a_i\rangle \quad \rightarrow \quad \hat{U}_A |\Psi\rangle = \sum c_i \hat{U}_A |a_i\rangle$

$$= \sum c_i |x_i\rangle$$

and $p_i = |c_i|^2 = |\langle a_i | \Psi\rangle|^2$ the general Born rule

The complex detector "measures" all operators of the form

$$\hat{A} = \sum \lambda_i |a_i\rangle\langle a_i|$$

Conclusions:

- Entropic Dynamics provides an alternative to Action Principles.
- No need for quantum probabilities or logic.
- The t in the Laws of Physics is entropic time.
- Quantum measurement is just an exercise in inference.
- Unitary evolution and wave function collapse correspond to infinitesimal and discrete updating.
- Position is “real”. Other observables are not. They are “created” by the act of measurement.

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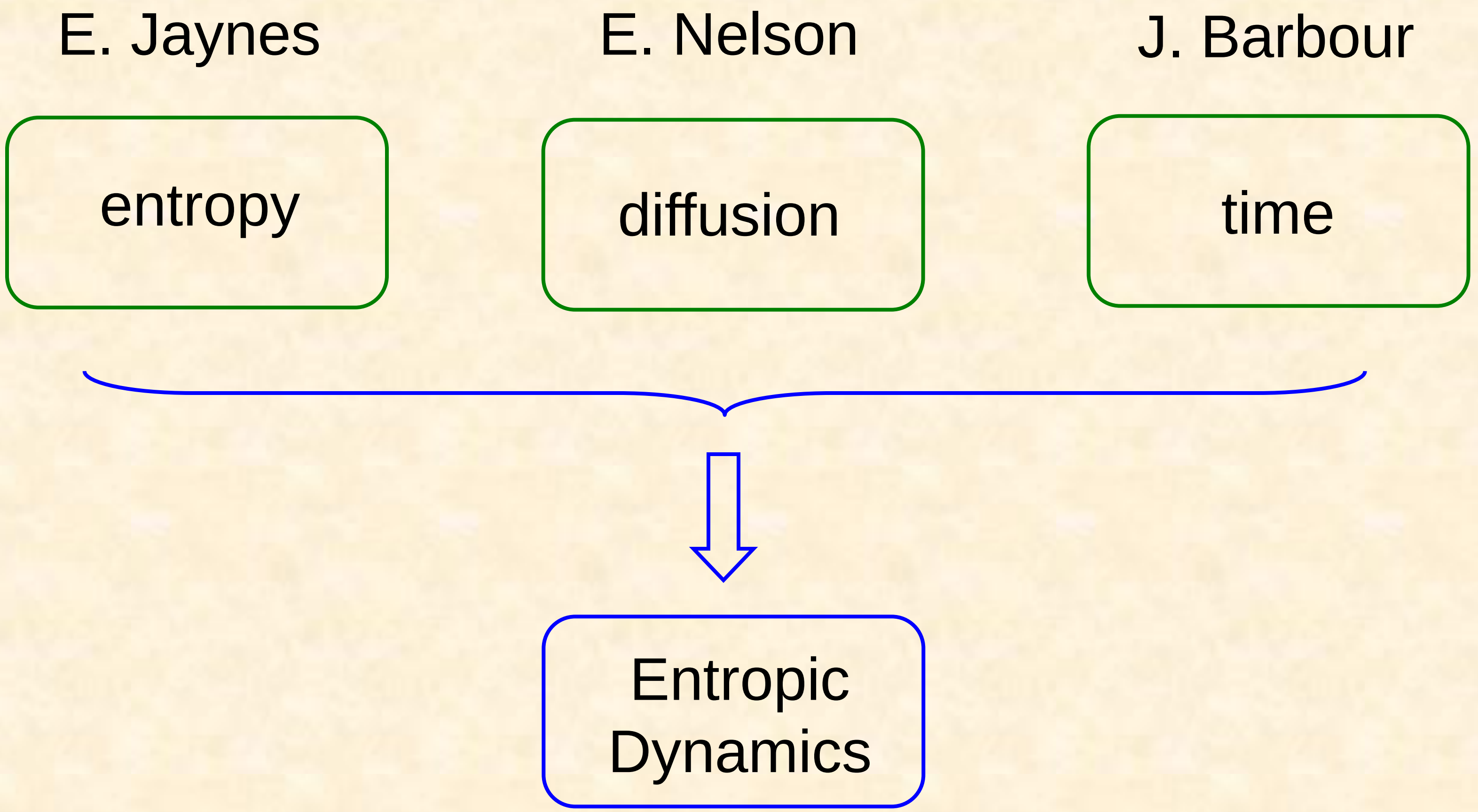
Three ingredients:



Entropic inference:

the prior $q(x)$ — the posterior $p(x)$

To update from old beliefs to new beliefs when new information becomes available.

the constraints

Amplification from micro to macro scales

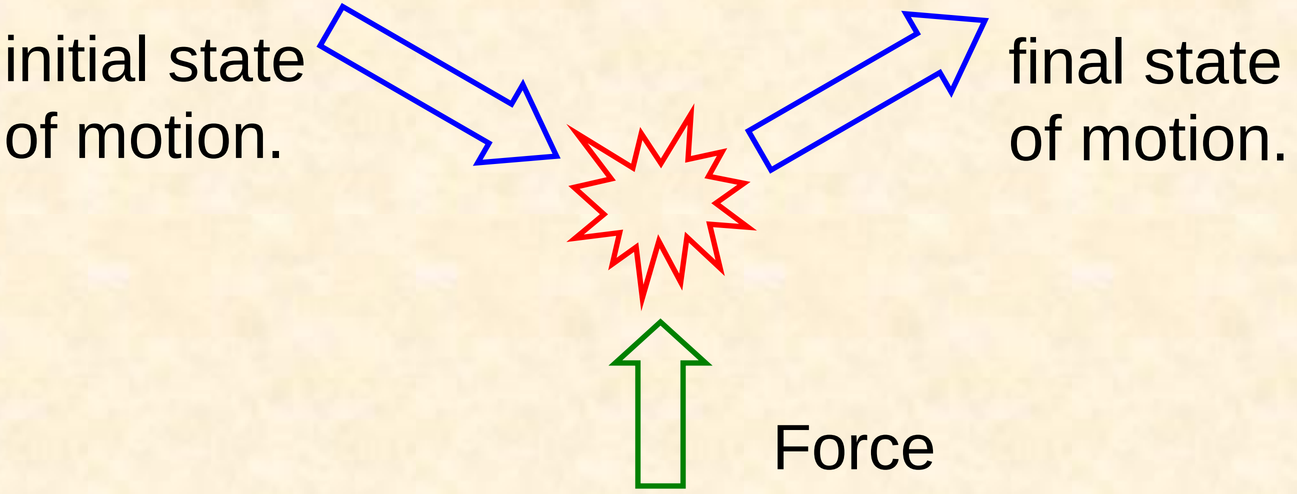
The amplifier is prepared in an unstable state.

When $|a_i\rangle \rightarrow \hat{U}_A |a_i\rangle = |x_i\rangle$ the amplifier jumps to state α_r.

The problem is to infer x_i from α_r.

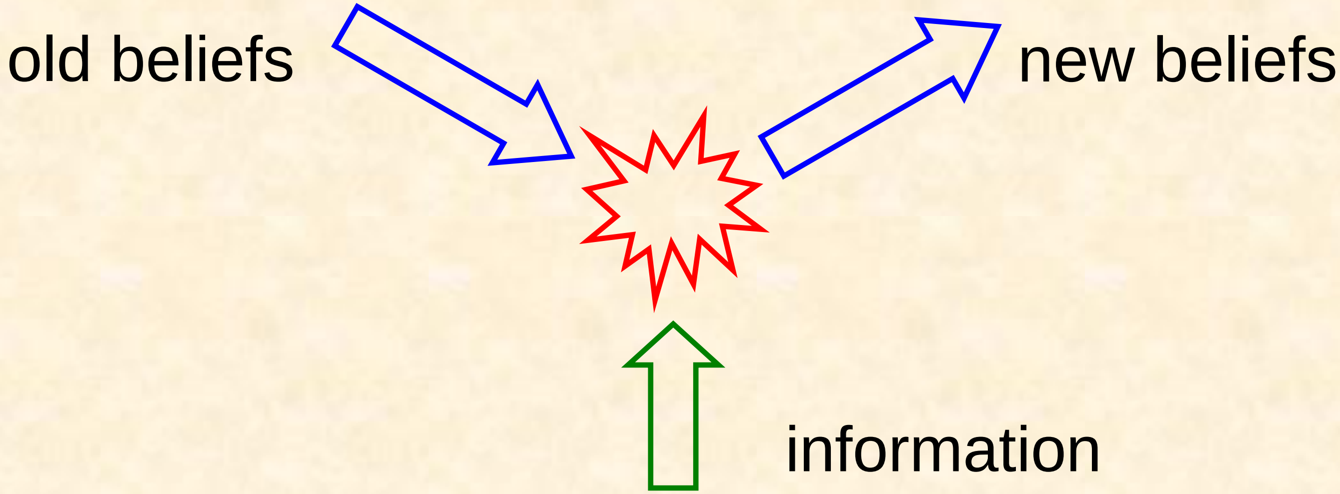
Use Bayes' rule: $$P(x_i \mid \alpha_r) = P(x_i)\frac{P(\alpha_r \mid x_i)}{P(\alpha_r)}$$

An analogy from physics:



Force is whatever induces a change of motion: $\vec{F} = \dfrac{\Delta \vec{p}}{\Delta t}$

Inference is dynamics too!



Information is what induces the change in rational beliefs.

Information is what constrains rational beliefs.

The quantum measurement problem:

The sources of all quantum weirdness: { indeterminism / superposition }

What is the source of indeterminism?

Why don't we see macroscopic entanglement?

How do we get definite outcomes?

Are observed values created by measurement?

(2) Instants are ordered: the Arrow of Entropic Time

A time-reversed evolution:

$$\rho(x,t) = \int dx' \, P(x \mid x') \rho(x',t')$$

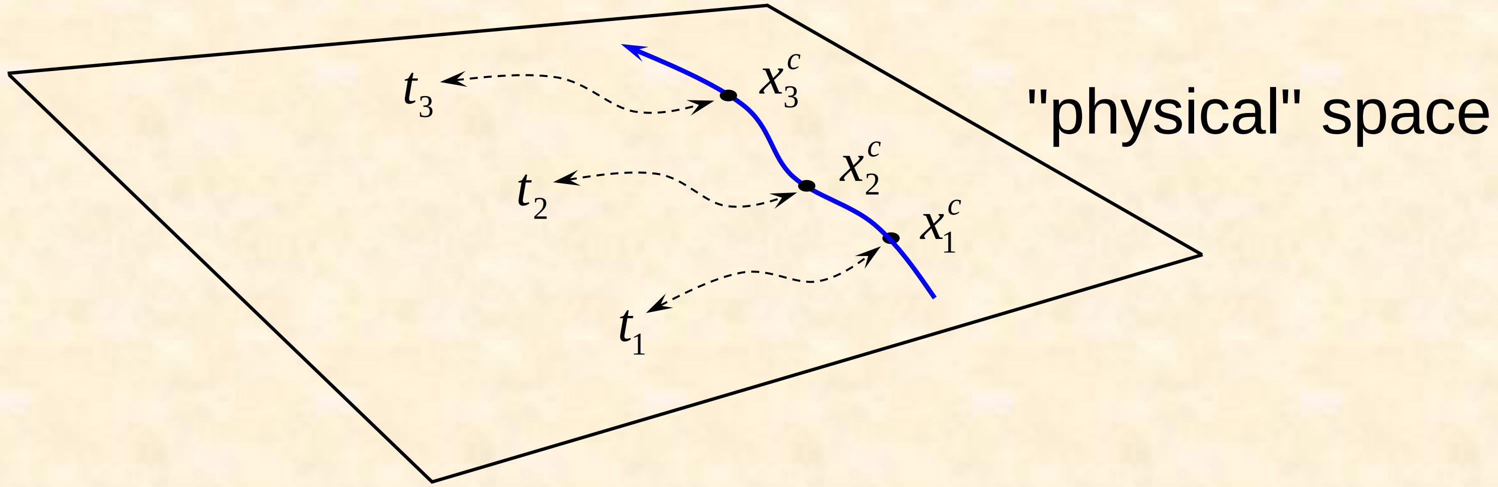
Bayes' theorem $$P(x \mid x') = \frac{P(x)}{P(x')} P(x' \mid x)$$

There is no symmetry between prior and posterior.

Entropic time only goes “forward”. Configuration space

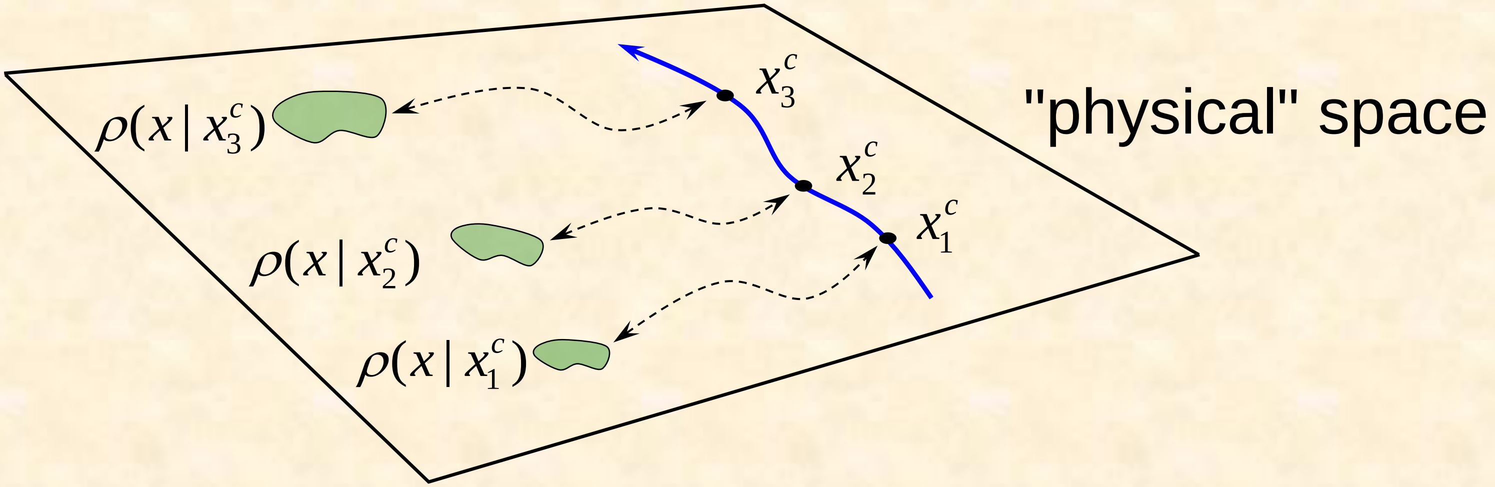
More on entropic time

A clock follows a classical trajectory.



More on entropic time

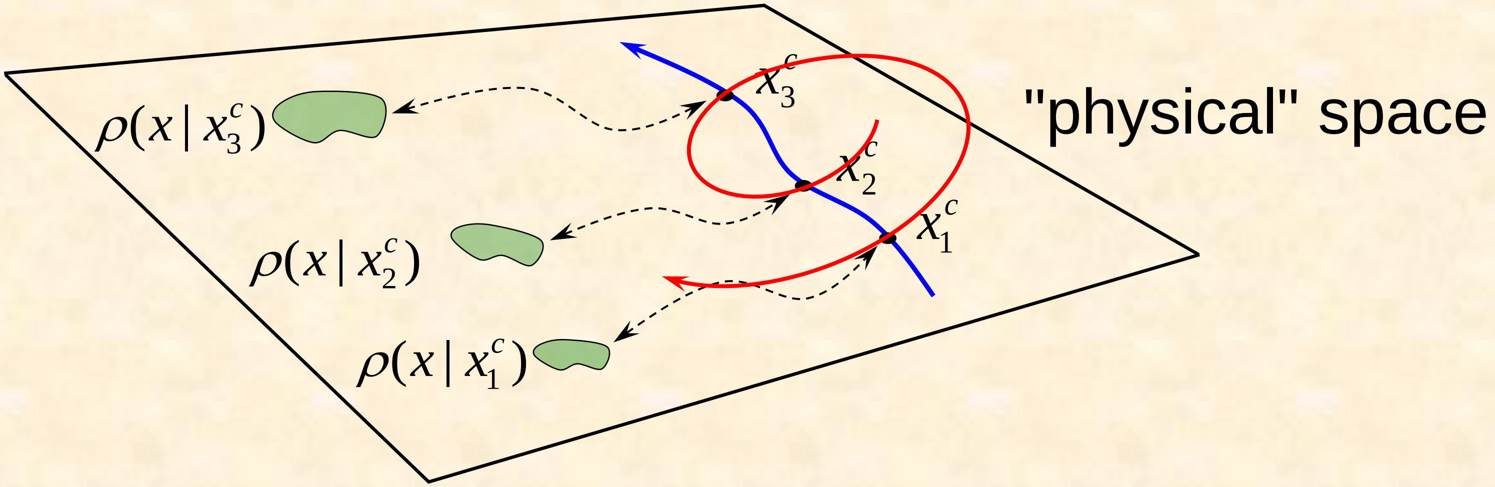
For the composite system of particle and clock:



Entropic time *vs* "physical" time?

We observe correlations at an instant.

We do not observe the "absolute" order of the instants.



Entropic time is all we need.

There is an arrow of entropic time.