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Spin and charge from space and time

in cooperation with

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Charge and Spin

- Charge
  - feels Coulomb and Lorentz forces,
  - is quantised, $e_0 = 1.602 \cdot 10^{-19}$ C,
    Faraday's laws of electrolysis, J. J. Thomson 1897, Millikan 1908,
  - NO electrodynamical and quantum mechanical reason to be quantised.
- Spin $\mathbf{s}$
  - no classical analogue,
  - 2 spin states of electron, spin up and down,
  - group SU(2), like isospin,
  - $\mathbf{j} = \mathbf{l} + \mathbf{s}$,
    spin $\mathbf{s}$ contributes to angular momentum,
    spin has some relation to space,
    isospin is only internal degree of freedom.

Space-Time

- special relativity
  - space and time are related,
  - no absolute space
- gravitation theory
  - space-time is not rigid,
  - space-time is deformed by matter and energy density,
  - length scales are coordinate dependent,
  - described by metric $g_{\mu\nu}$ in gravitational theory,
  - by local translations in Poincarè gauge theory.

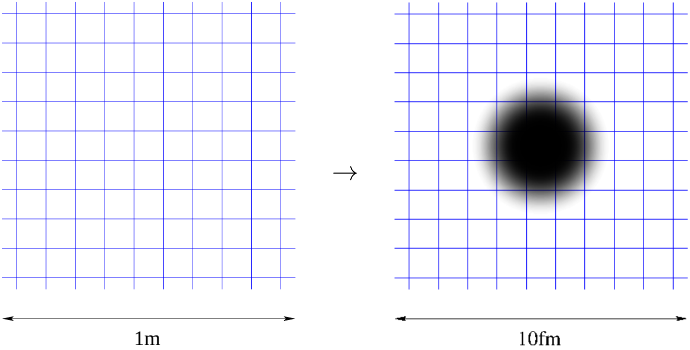
Charges – properties of Space?

Let us try a model $\cdots$

Imagine $\cdots$

flat Minkowski space

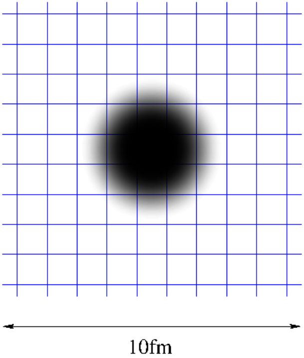
at macroscopic distances



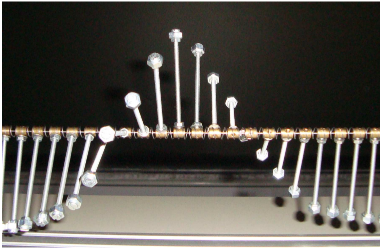
dislocations at microscopic distances

$\cdots$ **"dislocations" by local rotations of space.**

local rotations of Space



$\longleftrightarrow$



: mechanics: Peter Pataki foto: Gerald Pechoc

by rotating Dreibein (Triade) in space.

Rotation by 2π in every direction.

Describe rotations of Dreibein

- Quaternions $Q = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$: $\quad \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$
  - Rodriguez 1840 (Annales de Gergonne)
  - Hamilton 1843
- SO(3)-group: $2\pi = 4\pi$-rotation,
- SU(2)-group: $2\pi \neq 4\pi$-rotation,
  - SU(2) double covering group of SO(3),

We use SU(2)- description

$$q_0^2 + \vec{q}^2 = 1,\ \vec{q} = (q_1, q_2, q_3),\ q_0 = \cos\alpha,\ \vec{q} = \vec{n}\sin\alpha,\ \vec{n}^2 = 1.$$

$$(\mathbf{i}, \mathbf{j}, \mathbf{k}) = -\mathrm{i}\vec{\sigma},\ , \quad x^\mu = (ct, \mathbf{r}).$$

$$Q(x) = \mathrm{e}^{-\mathrm{i}\alpha(x)\vec{\sigma}\vec{n}(x)} = \cos\alpha(x) - \mathrm{i}\vec{\sigma}\vec{n}(x)\sin\alpha(x),$$

Field configurations $\pm Q(\mathbf{r})$ are identical, 3 degrees of freedom
SU(2) isomorphic to S^3, the sphere in 4 dimensions.

Describe geometry of S^3

Curves in space-time, parametrised by s and t: $x^\mu(s), x^\mu(t)$
vector field: $(\partial_s Q)\, Q^\dagger = -\mathrm{i}\vec{\sigma}\vec{\Gamma}_s$, connection one-form,

$$\vec{\Gamma}_s = \dot{\alpha}\vec{n} + \sin\alpha\cos\alpha\,\dot{\vec{n}} + \sin^2\alpha\,\vec{n}\times\dot{\vec{n}}$$

area density: $\vec{R}_{st} = \vec{\Gamma}_s \times \vec{\Gamma}_t$, connection two-form (curvature).

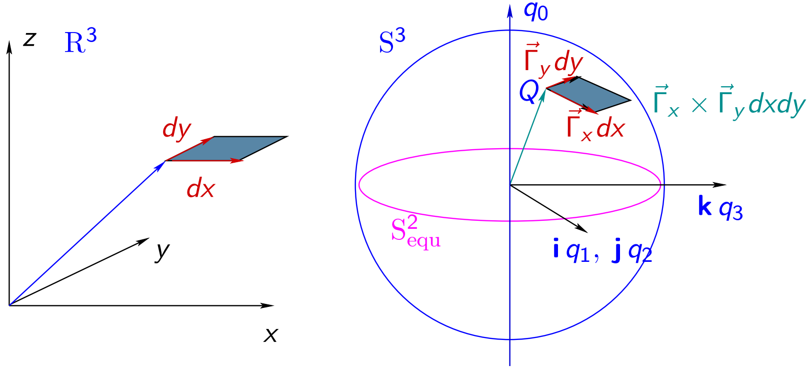


Figure: map: $dx \times dy \mapsto \vec{\Gamma}_x \times \vec{\Gamma}_y dxdy$

Relate geometry to physics

- area density (curvature) $\mapsto$ field strength:

$$\vec{R}_{xy} = \vec{\Gamma}_x \times \vec{\Gamma}_y = \frac{\vec{\Gamma}_x \times \vec{\Gamma}_y dxdy}{dxdy} \;\mapsto\; \vec{E}_z = \frac{e_0}{4\pi\epsilon_0}\vec{R}_{xy},$$

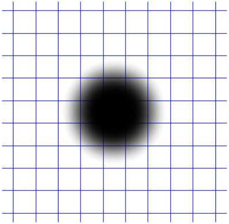
$$c\vec{B}_z = \frac{e_0}{4\pi\epsilon_0}\vec{R}_{tz}$$

$$^\star\vec{F}_{\mu\nu} = \frac{e_0}{4\pi\epsilon_0}\vec{R}_{\mu\nu}$$

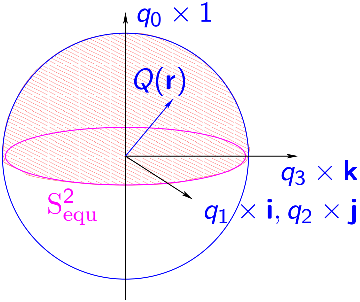
$$\vec{R}_{\mu\nu} = \partial_\mu\vec{\Gamma}_\nu - \partial_\nu\vec{\Gamma}_\mu - \vec{\Gamma}_\mu \times \vec{\Gamma}_\nu$$

- energy density $\mapsto$ (field strength)^2,
- action density: $\mathcal{L}_e = -\frac{\alpha_f \hbar c}{4\pi}\frac{1}{4}\,\vec{R}_{\mu\nu}\vec{R}^{\mu\nu}$

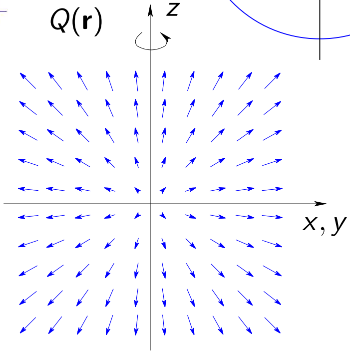
Static elementary Charge



$R^3 \quad \mapsto \quad S^3$



$Q(\mathbf{r}) : R^3 \mapsto S^3.$



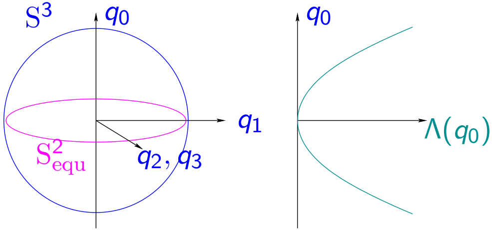
unit charge covers hemisphere of S^3,

$\vec{q}(\mathbf{r}) = \sin\alpha(\mathbf{r})\vec{n}(\mathbf{r}).$

Potential: cosmological function

action density: $\mathcal{L}_e = -\frac{\alpha_f \hbar c}{4\pi} \frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu}$, Soliton dissolving, not yet stable

We need: Equilibrium between compressing and broadening



Potential term $\Lambda(q_0)$ with minimum on $\mathrm{S}^2_{\mathrm{equ}}$ of S^3.

Compressing term: $\Lambda(q_0) = \frac{q_0^{2m}}{r_0^4},\ m = 1, 2, 3, \cdots$

Lagrangian: $\mathcal{L} = \mathcal{L}_e - \mathcal{H}_p = -\frac{\alpha_f \hbar c}{4\pi} \left(\frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \Lambda \right)$

Consequences: two-dimensional degeneracy of vacuum,
scale r_0, $\rho = r/r_0$

Stable minima of energy (Solitons)

- hedgehog ansatz: $\vec{n}(x) = \frac{\vec{r}}{r}$, $\qquad x = (ct, \mathbf{r})$

  $Q(x) = \cos\alpha(x) + \mathrm{i}\vec{\sigma}\vec{n}(x)\sin\alpha(x), \quad \text{with} \quad \alpha = \alpha(\rho), \qquad \rho = r/r_0$

- minimisation of energy leads to non–linear differential equation

$$\partial_\rho^2 \cos\alpha + \frac{(1-\cos^2\alpha)\cos\alpha}{\rho^2} - m\rho^2\cos^{2m-1}\alpha = 0$$

- solution for $m = 3$

$$\alpha(\rho) = \mathrm{atan}(\rho).$$

- energy of soliton

$$E = \frac{\alpha_f \hbar c}{r_0}\frac{\pi}{4} \quad \text{with} \quad \alpha_f \hbar c = 1.44 \text{ MeV fm}.$$

- compare with electron

$$m_e c^2 = 0.511 \text{ MeV}, \text{ we get } r_0 = 2.21 \text{ fm}.$$

General 4-dimensional formulation

- defined by Lagrangian $\mathcal{L} = -\frac{\alpha_f \hbar c}{4\pi} \left(\frac{1}{4} \vec{R}_{\mu\nu} \vec{R}^{\mu\nu} + \Lambda \right)$,
- general equations of motion,
- relativistic dynamics of arbitrary many-soliton configurations, dynamics of solitons and waves,
- differences to Maxwell electrodynamics:
  - (+) charges and fields (non) separated, $Q(x) \leftrightarrow A^\mu(x), j^\mu(\mathrm{x})$,
  - (+) numbers of degrees of freedom: $3 \leftrightarrow 8$,
  - (+) (non) integer multiples of elementary charge e_0,
  - (+) (in)finite self energy of charges,
  - (+) charges extended (point-like) objects,
  - (?) non-vanishing magnetic currents (non-solitonic),
  - (?) α-waves.

Separate charges and electro-magnetic fields

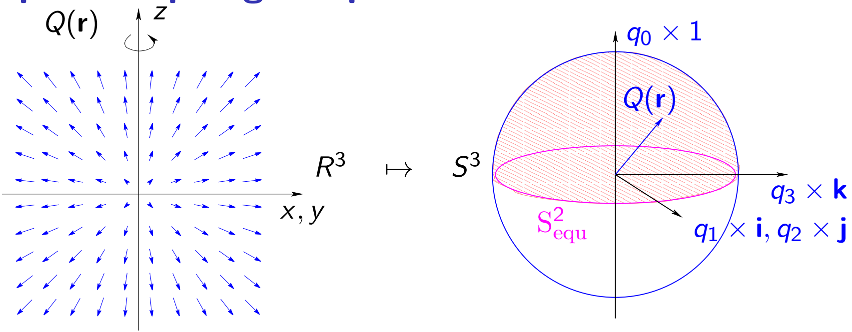
large distance behaviour: $r \gg r_0$, electro-dynamic limit, Dirac limit

- Maxwell equations
- Coulomb and Lorentz forces
- U(1) gauge symmetry (rotation of Dreibein around $\vec{n}$)
- two massless excitations: photons as Goldstone bosons
  topological quantum number $S^3 \mapsto S^2$, Hopf index,
  possibly: Hopf index = photon number

Conjectures:

- particles are solitons,
- only particles can be detected,
- waves escape our detectors, they can't be caught,
  α-waves, magnetic currents, non-solitonic $\vec{n}$-waves,
  contributions to dark matter and dark energy???

Spin, a topological quantum number



Field configuration $Q(\mathbf{r})$ of unit charge covers hemisphere of S^3, $s = \frac{1}{2}$.

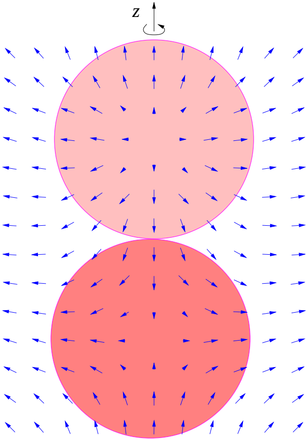
Spin quantum number s

$$s = \left| \frac{1}{V(\mathbb{S}^3)} \int_0^\infty \mathrm{d}r \int_0^\pi \mathrm{d}\vartheta \int_0^{2\pi} \mathrm{d}\varphi \, \vec{\Gamma}_r (\vec{\Gamma}_\vartheta \times \vec{\Gamma}_\varphi) \right| \tag{1}$$

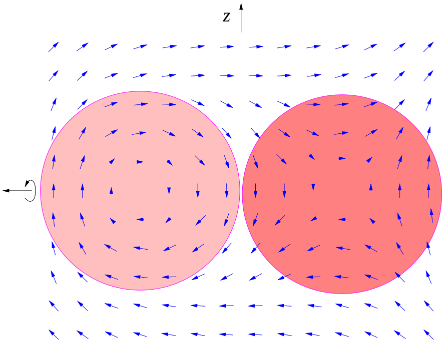
Magnetic quantum numbers $m_s = \pm 1/2$: Upper and lower hemisphere

Spin, an angular momentum

Symmetry broken vacuum, $Q(\infty) = -\mathrm{i}\sigma_3$, Field at infinity is constant, No rigid rotation possible,



$S = 0$, Charge Zero



$j = l - s$

Emerging quantum mechanics

where is room for quantum mechanics?
Quantum fluctuations:

- particles are disturbed on their classical path,
- by waves propagating with velocity of light,
- no special reference frame,
- average momentum is unchanged by such perturbations

Interference:

- solitons are extended objects and possibly get in resonance with waves,
- interference by Couder's mechanism.

Summary

Conclusions

- Only Space and Time.
- Only 3 rotational degrees of freedom of space were used.
- Charges can be described by 2π-rotations of space.
- Spin angular momentum as Eigen-angular momentum,
  Spin as a consequence of orbital motion.

Conjectures

- Stable particles are stable solitons with topological quantum numbers.
- Only particles can be detected.
- Waves escape our detectors.
- Waves disturb the pathes of particles -> Quantum Mechanics
- Particles get in resonance with waves -> interference
  (in analogy to Couder's experiments)

Aftermath

Einstein: Physics should be as simple as possible.

I think: Physics is geometry and not algebra.
We should use algebra only to describe the geometry.

Thanks