Quantum Probabilities from Classical Random Fields and Detectors of Threshold Type

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Close

QM is an approximation (very good one) of another theory, namely, theory of classical random fields.

Soon or later we shall show experimentally that laws of QM are approximative.

The basic law of QM is the Born's rule. Its violation is predicted by my model.

Khrennikov A.: Detection Model Based on Representation of Quantum Particles by Classical Random Fields: Born's Rule and Beyond. Found. Physics 39, 997-1022 (2009).

Present experimental studies on violation of Born's rule: Gregor Weihs testing for three slit experiment. It is based on R. Sorkin's work.

G 't Hooft: subquantum theory will be very simple.

Zeilinger (private conversation): It will be so exotic that those guys who nowadays struggle against QM will dream about those good days of traditional QM.

My model is as simple as possible – just comeback to classical random fields. May be too simple even for majority of those who question the conventional interpretation of QM.

Where is quantumness in classical field theory? **In detectors of the threshold type!** (see also A. Lande, W. Lamb).

Where is noncommutativity? In quadratic forms of classical fields.

Do fields have objective properties? Yes and no.

"Photon"interpreted as a classical pulse objectively has electric and magnetic components of its field, but does a wave has position?

If one uses detectors of the threshold type, wave's position depends on the placement of detectors and thresholds used in them.

Malus's Law:

According to Malus, when completely plane polarized light is incident on the analyzer, the intensity I of the light transmitted by the analyzer is directly proportional to the square of the cosine of angle between the transmission axes of the analyzer and the polarizer.

Born's rule via discretization based on the threshold type detectors. However, discretization is an approximation therefore "Malus law for clicks" = Born's rule only approximately accounts the numbers of clicks in corresponding channels.

Discrimination threshold

A lot of noise is involved in the process of detection.

An important source of noise is the *multiplication process*. For example, in photo-multipliers, once an electron has been extracted from the metal, it is accelerated in vacuum by an electric field until its kinetic energy is enough to extract other bounds electrons (secondary emission) when striking the surface of another metal surface (dynode), which will in turn be used accelerated onto other dynodes to free more and more electrons, until that flow of electrons becomes measurable as anodic current.

The form and energy of output spikes vary significantly from one liberated energy carrier to another. Thus the **gain is a random variable.**

Besides of noise produced by detectors, noise responsible for so called **dark counts** plays an important role. This noise of the random background is inescapable. In **average** spikes corresponding to signal detections differ in the amplitude from noise generated spikes.

This fact provides a possibility to filter noise generated spikes by using a **discrimination threshold,** denote the later by the symbol $\boldsymbol{E_d}$.

The selection of this threshold is a delicate procedure. By selecting too low $\boldsymbol{E_d}$ experimenter would count to many noise generated spikes, in particular, dark counts. By selecting it too high experimenter would discard too many spikes generated by the signal. In both cases quantum statistics would be essentially disturbed.

Threshold detection scheme

We consider a threshold type detector with the threshold $\boldsymbol{E_d}$. It interacts with a random field $\boldsymbol{\phi(s,\omega)}$, where $\boldsymbol{s}$ is time and $\boldsymbol{\omega}$ is a chance parameter describing randomness. For a moment, we consider the $\mathbf{C}$-valued random field (complex stochastic process).

The energy of the field is given by

$$\boldsymbol{E(s,\omega) = |\phi(s,\omega)|^2}$$

(hence, the random field has the physical dimension $\sim \sqrt{\mathbf{energy}}$).

A threshold detector clicks at the first moment of time $\boldsymbol{\tau(\omega)}$ (the first hitting time) when signal's energy $\boldsymbol{E}$ multiplied by the gain $\boldsymbol{g}$ exceeds the threshold:

(1)

$$\boldsymbol{gE(\tau(\omega),\omega) \geq E_d, \text{i.e.}, \tau(\omega) = \inf\{\mathrm{s} \geq 0 : \mathrm{gE}(\mathrm{s},\omega) \geq \mathrm{E_d}\}.}$$

PDF of the detection moment

We proceed under the following basic assumption. After arriving to a threshold type detector a classical signal (random field) behaves inside this detector as the (complex) **Brownian motion,** i.e., the $\phi(s, \omega)$ is simply the *Wiener process*, the Gaussian process having zero average at any moment of time

$$\overline{\phi(s, \omega)} = 0. \tag{2}$$

and the covariance function

$$\overline{\phi(s_1, \omega)\phi^*(s_2, \omega)} = \min(s_1, s_2)\sigma^2; \tag{3}$$

in particular, we can find average of its energy

$$\overline{E(s, \omega)} = \sigma^2 s. \tag{4}$$

From this equation, we see that the coefficient σ^2 has the physical dimension of **power.**

We are interested in the probability distribution of the moments of the E_d-threshold detection for the energy of the Brownian motion. Since moments of detection are defined formally as hitting times, we can apply theory of hitting times. Consider

(5)
$$\tau_a(\omega) = \inf\{s \geq 0 : E(s,\omega) \geq a^2\} = \inf\{s \geq 0 : |\phi(s,\omega)| \geq a\}.$$

Its probability distribution function (PDF) is given by the complicated expression, see, e.g., Shyryaev

$$P(\tau_a \leq \Delta t) = 4\sum_{k=0}^{\infty}(-1)^k\Big[1 - \Phi\Big(\frac{a(1+2k)}{\sqrt{\sigma^2\Delta t}}\Big)\Big], \tag{6}$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-u^2/2}du$$

is the PDF of the standard Gaussian distribution.

The **complementary error function**:

$$\mathbf{erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du.}$$

Now we select $\boldsymbol{a = \sqrt{\frac{E_d}{g}}}$ and set $\boldsymbol{\tau \equiv \tau_a}$. We obtain:

$$(7) \quad \boldsymbol{P(\tau(\omega) \le \Delta t) = 2 \sum_{k=0}^{\infty} (-1)^k \mathrm{erfc}\left((1 + 2k)\sqrt{\frac{E_d}{2\sigma^2 \Delta t g}}\right).}$$

Asymptotics for the detection moment

In coming considerations, Δt is the **average duration of the interaction of a signal with a threshold detector.** The experimental scheme can be described in the following way. There is a source of random pulses of e.g. classical electromagnetic field. (Such pulses can be identified with wave packets used in the quantum formalism.) Each pulse propagates in space and finally arrives to a detector. The aforementioned temporal parameter Δt is the (average) duration of interaction of an input pulse with a detector. The quantity

$$\overline{E}_{\text{pulse}} = \sigma^2 \Delta t \tag{8}$$

is the average energy of emitted pulses.

In fact, this is the average energy which is transmitted to a detector by a pulse in the process of interaction. We proceed under the assumption that there are no losses and a detector "eats" (in average) the total energy of the emitted pulse.

We shall proceed under the basic assumption that this energy is essentially less than the threshold, i.e.,

$$\epsilon \equiv \frac{\overline{E}_{\text{pulse}}}{E_d} << 1. \tag{9}$$

This is a realistic assumption, since the discrimination threshold is set for the amplified signals from the detector and the gain producing this amplification is very large. In reality $\epsilon \sim 10^{-7}$.

Additional randomization by using a random gain

As was pointed out, the gain is by itself a random variable:

$$\tau(\omega) = \inf\{s \geq 0 : g(\omega)E(s,\omega) \geq E_d\}.$$

Born's rule for the detection probability

Consider now a random signal $\phi(s, \omega)$ valued in the m-dimensional complex HIlbert space H, where m can be equal to infinity. Let (e_j) be an orthonormal basis in H. The vector-valued (classical) signal $\phi(s, \omega)$ can be expanded with respect to this basis, **"Malus law":**

$$(10) \qquad \phi(s, \omega) = \sum_j \phi_j(s, \omega) e_j, \; \phi_j(s, \omega) = \langle e_j, \phi(s, \omega) \rangle.$$

This mathematical operation is physically realized as splitting of the signal $\phi(s, \omega)$ into components $\phi_j(s, \omega)$. These components can be processed through mutually disjoint channels, $j = 1, 2, ...m$. We now assume that there is a threshold detector in each channel, $D_1, ..., D_m$. We also assume that all detectors have the same threshold $E_d > 0$ and the same probability distribution of the gain, with the density $\rho_g(\lambda)$.

Suppose now that $\phi(s,\omega)$ is the H-valued Brownian motion (the Wiener process in H). This process is determined by the covariance operator $B : H \to H$. **Any covariance operator is Hermitian, positive, and the trace-class and vice versa.** The complex Wiener process is characterized by the Hermitian covariance operator. (We remark that complex-valued random signals are widely used in e.g. radio-physics.) We have, for $y \in H$,

$$\overline{\langle y, \phi(s,\omega)\rangle} = 0,$$

and, for $y_j \in H, j = 1, 2,$

$$\overline{\langle y_1, \phi(s_1,\omega)\rangle\langle \phi(s_2,\omega), y_2\rangle} = \min(s_1, s_2)\langle By_1, y_2\rangle.$$

The latter is the covariance function of the stochastic process; in the operator form: $B(s_1, s_2) = \min(s_1, s_2)B$.

We note that the dispersion of the $\boldsymbol{H}$-valued Wiener process (at the instant of time $\boldsymbol{s}$) is given by

$$\boldsymbol{\Sigma_s^2 \equiv \overline{\|\phi(s,\omega)\|^2} = s\mathrm{Tr}B.} \tag{11}$$

This is the average energy of this random signal at the instant of time $\boldsymbol{s}$. Hence, the quantity

$$\boldsymbol{\Sigma^2 = \frac{\Sigma_s^2}{s} = \mathrm{Tr}B} \tag{12}$$

has the physical dimension of power; this is the average power of the signal of the Brownian motion type (it does not depend on time).

We also remark that by normalization of the covariance function for the fixed $\boldsymbol{s}$ by the dispersion we obtain the operator,

$$\boldsymbol{\rho = B/\mathrm{Tr}\mathbf{B},} \tag{13}$$

which formally has all properties of the **density operator** used in quantum theory to represent quantum states. Its matrix elements have the form

$$\boldsymbol{\rho_{ij} = b_{ij}/\Sigma^2.} \tag{14}$$

These are dimensionless quantities. The relation (13) plays a fundamental role in our approach : **each classical random process generates a quantum state (in general mixed) which is given by the normalized covariance operator of the process.**

‘t Hooft: can quantum and classical models be mapped one to another?

Can theory of classical random fields be represented in terms of quantum theory?

The answer is yes! Covariance operators under the natural normalization give us "density operators quadratic forms of random fields are mapped onto "quantum observables"given by Hermitian operators representing these forms.

Khrennikov A.: Entanglement's dynamics from classical stochastic process. Europhysics Letters 88, 40005.1-6 (2009)

Khrennikov A.: Quantum correlations from classical Gaussian correlations. J. Russian Laser Research 30, 472-479 (2009)

We now consider components of the random signal $\phi(s,\omega)$ and their correlations:

(15)
$$\overline{\phi_i(s_1,\omega)\phi_i^*(s_2,\omega)} = \min(s_1,s_2)\langle Be_i, e_j\rangle = \min(s_1,s_2) b_{ij}.$$

In particular,

$$\sigma_j^2(s) \equiv \overline{E_j(s,\omega)} \equiv \overline{|\phi_j(s,\omega)|^2} = s\, b_{jj}. \tag{16}$$

This is the average energy of the jth component at the instant of time s. We also consider its average power:

$$\sigma_j^2 = b_{jj}. \tag{17}$$

We remark that the average power of the total random signal is equal to the sum of powers of its components:

$$\Sigma^2 = \sum_j \sigma_j^2. \tag{18}$$

We have

$$N_j = \frac{T}{\Delta t} \int d\lambda \rho_g(\lambda) P(\tau_j \leq \Delta t | g = \lambda)$$

$$= \frac{2T}{\Delta t} \sum_{k=0}^{\infty} (-1)^k \int d\lambda \rho_g(\lambda) \mathrm{erfc}\Big((1 + 2\mathrm{k})\sqrt{\frac{\mathrm{E_d}}{2\lambda\sigma_{\mathrm{j}}^2 \Delta \mathrm{t}}}\Big). \tag{19}$$

The total number of clicks in all detectors is the sum of N_j :

$$N = \frac{2T}{\Delta t} \sum_{k=0}^{\infty} (-1)^k \int d\lambda \rho_g(\lambda) \sum_j \mathrm{erfc}\Big((1 + 2\mathrm{k})\sqrt{\frac{\mathrm{E_d}}{2\lambda\sigma_{\mathrm{j}}^2 \Delta \mathrm{t}}}\Big). \tag{20}$$

Hence, the probability of a click in the jth detector is given by sufficiently complex formula (generalized Born's rule):

(21)

$$P_j = \frac{\sum_{k=0}^{\infty}(-1)^k \int d\lambda \rho_g(\lambda) \mathrm{erfc}\Big((1+2\mathrm{k})\sqrt{\frac{\mathrm{E_d}}{2\lambda\sigma_{\mathrm{j}}^2\Delta \mathrm{t}}}\Big)}{\sum_{k=0}^{\infty}(-1)^k \int d\lambda \rho_g(\lambda) \sum_j \mathrm{erfc}\Big((1+2\mathrm{k})\sqrt{\frac{\mathrm{E_d}}{2\lambda\sigma_{\mathrm{j}}^2\Delta \mathrm{t}}}\Big)}.$$

$$P_j = \frac{\sum_{k=0}^{\infty}(-1)^k \int d\lambda \rho_g(\lambda) \mathrm{erfc}\Big((1+2\mathrm{k})\sqrt{\frac{1}{2\lambda\epsilon}}\Big)}{\sum_{k=0}^{\infty}(-1)^k \int d\lambda \rho_g(\lambda) \sum_j \mathrm{erfc}\Big((1+2\mathrm{k})\sqrt{\frac{1}{2\lambda\epsilon}}\Big)}. \quad (22)$$

We now recall that want to find the asymptotics for

$$\epsilon \equiv \frac{\overline{E}_{\mathrm{pulse}}}{E_d} << 1. \quad (23)$$

$$\text{(24)} \qquad \lim_{\epsilon \to 0} P_j(\epsilon) = \frac{\sigma_j^2}{\Sigma^2} = \rho_{jj} = \mathrm{Tr}\rho\hat{\mathrm{C}}_{\mathrm{j}},$$

where the projection operator $\hat{C}_j = |e_j\rangle\langle e_j|$ on the vector e_j :

A. Khrennikov, Violation of Bell's inequality by correlations of classical random signals. *Physica Scripta*, **T151**, art. number 014003 (2012)

A. Khrennikov, Quantum probabilities and violation of CHSH-inequality from classical random signals and threshold type detection scheme. *Prog. Theor. Phys.* **128**, No. 1, 31-58 (2012).

arxiv.org/abs/1111.1907

Next Växjö-conference on quantum foundations, June 9-12, 2014,

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