# Weak Values, Bohmian Mechanics, and the Many Interacting Worlds Approach to QM

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# Outline

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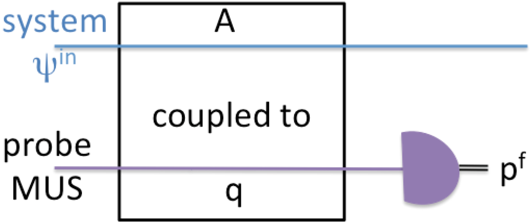
# How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

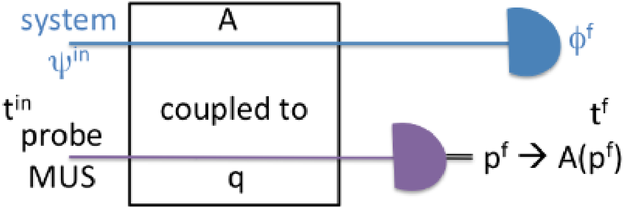
- PRL **60**, 1351 (1988).
- Consider an arbitrary system observable $A$.
- Assume a probe with $[\hat{q}, \hat{p}] = i$, initially in a MUS (minimum uncertainty state).




- The probe state is defined by $\sigma_p^{\rm in}$, $\bar{p}^{\rm in}$, and $\bar{q}^{\rm in} = 0$.
- Assume (von Neumann) $\hat{H} = \delta(t)\hat{A} \otimes \hat{q}$, so that $\hat{p}^{\rm f} - \hat{p}^{\rm in} = \hat{A}$.
- By measuring $p^{\rm f}$ we can **estimate** $A$ as $A(p^{\rm f}) = p^{\rm f} - \bar{p}^{\rm in}$.

# Initial and Final States.

- For initial system state $|\psi^{\rm in}\rangle$, we can obtain, by repeating the experiment,
$$\mathrm{E}[A(p^{\rm f})|\psi^{\rm in}] = \langle\psi^{\rm in}|\hat{A}|\psi^{\rm in}\rangle.$$
- Now consider a final *strong* measurement on the system too.




- Consider the sub-ensemble where the final result corresponds to projecting onto state $|\phi^{\rm f}\rangle$.
- Then we can consider the *post-selected* average $\mathrm{E}[A(p^{\rm f})|\psi^{\rm in}, \phi^{\rm f}]$.

# The Weak Measurement Limit

- In the **weak measurement limit**, $\sigma_p \to \infty$,

$$\mathrm{E}[A(p^{\mathrm{f}})|\psi^{\mathrm{in}},\phi^{\mathrm{f}}] \to {}_{\phi^{\mathrm{f}}}\langle A^{\mathrm{w}}\rangle_{\psi^{\mathrm{in}}} \equiv \mathrm{Re}\frac{\langle\phi^{\mathrm{f}}|\hat{A}|\psi^{\mathrm{in}}\rangle}{\langle\phi^{\mathrm{f}}|\psi^{\mathrm{in}}\rangle}.$$

Q Why is this the weak measurement limit?

A Because very little information in any individual result

$$A(\hat{p}^{\mathrm{f}}) = \hat{A} + (\hat{p}^{\mathrm{in}} - \bar{p}^{\mathrm{in}})$$

and $\left\langle(\hat{p}^{\mathrm{in}} - \bar{p}^{\mathrm{in}})^2\right\rangle = \sigma_p^2 \to \infty$.

A Because weak (*not no*) disturbance:

$$\hat{s}^{\mathrm{f}} = \hat{s}^{\mathrm{in}} - i[\hat{s}^{\mathrm{in}},\hat{A}] \otimes \hat{q}^{\mathrm{in}}$$

and $\left\langle(\hat{q}^{\mathrm{in}})^2\right\rangle = 1/(2\sigma_p)^2 \to 0$ in this limit.

- Note: the weaker the measurement, the larger the number of repetitions required to obtain a reliable average.

# What (in my humble opinion) do Weak Values offer for "Fundamental Questions in Quantum Mechanics"?

Many FQiQM have no (or at least no unique) answers in standard QM, but in many cases, Weak Values **do** offer answers, which

- may be a new answer to the question,
- or single out one answer out of a (possibly infinite) set of answers that had been proposed,
- either of which may give new insights and prompt new research,
- and if not, at least they often enable an experiment to be done,
- which brings the issues to the attention of a broader audience (in a way that theory papers seldom do).

e.g. Superluminal tunnelling, measurement–disturbance relations, three box paradox, Cherenkov radiation in vacuum, Pre-jump oscillations in cavity QED, . . .

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# Outline

# A unique Bohmian velocity

- Consider a Bohmian world-configuration with equation of motion

$$\dot{\mathbf{x}} = \mathbf{v}_{\psi(t)}(\mathbf{x})$$

- There are infinitely many functional expressions for $\mathbf{v}_\bullet(\bullet)$ :

$$\partial P_{\psi(t)}(\mathbf{x})/\partial t + \nabla \cdot [P_{\psi(t)}(\mathbf{x};t)\mathbf{v}_{\psi(t)}(\mathbf{x})] = 0,$$

with $P_{\psi(t)}(\mathbf{x}) = \langle \psi(t)|\mathbf{x}\rangle\langle \mathbf{x}|\psi(t)\rangle$.

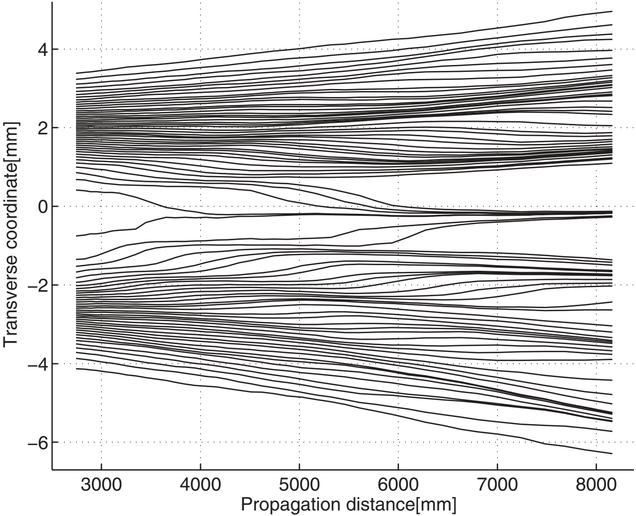
- But if we define (HMW, NJP, 2007)

$$\begin{aligned} \mathbf{v}_{\psi(t)}(\mathbf{x}) &= \lim_{\tau\to 0} \tau^{-1}\, \mathrm{E}_{\psi(t)}[\mathbf{x}_{\text{strong}}(t+\tau) - \mathbf{x}_{\text{weak}}(t)|\mathbf{x}_{\text{strong}}(t+\tau) = \mathbf{x}] \\ &= \lim_{\tau\to 0} \tau^{-1} \left[ \mathbf{x} - {}_{\langle \mathbf{x}|\hat{U}(\tau)}\langle \hat{\mathbf{x}}^{\mathrm{w}} \rangle_{|\psi(t)\rangle} \right]. \end{aligned}$$

one gets the standard Bohmian expression for $\mathbf{v}_{\psi(t)}(\mathbf{x})$ ...

# Experiment! Kocsis & *al.* & Steinberg (Science, 2011)

- ... and one can measure it (even as a "naive experimentalist")




Note that it is **not** possible to follow an individual particle.

These trajectories are created by patching together little increments inferred from the weak velocities.

# A unique Bohmian ontology?

- The weak-valued velocity formula evaluates in general to

$$\mathbf{v}_{\psi(t)}(\mathbf{x}) = \mathrm{Re}\frac{\langle\psi(t)|\mathbf{x}\rangle\langle\mathbf{x}|i[\hat{H},\hat{\mathbf{x}}]|\psi(t)\rangle}{\hbar\langle\psi(t)|\mathbf{x}\rangle\langle\mathbf{x}|\psi(t)\rangle}.$$

Q Is this always consistent with QM? i.e. Does

$$P_0(\mathbf{x}) = \langle\psi(0)|\mathbf{x}\rangle\langle\mathbf{x}|\psi(0)\rangle \to P_t(\mathbf{x}) = \langle\psi(t)|\mathbf{x}\rangle\langle\mathbf{x}|\psi(t)\rangle?$$

A Iff $\hat{H}$ is at most quadratic in operators canonically conjugate to $\hat{\mathbf{x}}$.

Q Isn't this a limitation of this approach?

A No! Because all physical Hamiltonians *are* so constrained *if we take* $\hat{\mathbf{x}}$ *to be the configuration operator* (as usual).
That is, this approach explains *why* HV $= \mathbf{x}$.

Q Does this prove that Bohmian mechanics is correct?

A Absolutely not. But it shows that it is self-substantiating, making it (I think) a very natural theory.

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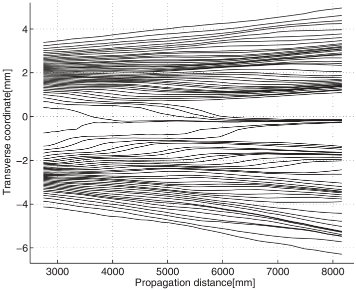
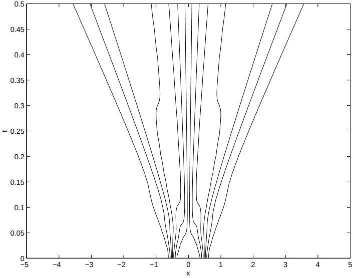
1. Weak Values
2. Measuring Bohmian-like trajectories
3. Many Interacting Worlds
   - One-dimensional Case
   - From Bohmian Mechanics to Many Interacting Worlds
4. Conclusions

# Repulsive trajectories?




- Suggestive of trajectories for a bunch of particles which repel one another.
- This is not to be taken literally.
- But why not?
- Schiff & Poirier, J. Chem. Phys. (2012) “Quantum Mechanics without Wavefunctions” developed this theory for a *continuous* ensemble of particles.
- Shortly after (and in ignorance of) this, we had a very similar idea.
- But we think it is difficult to imagine what a continuous ensemble means, so we are trying to develop the theory for a finite (but very large) ensembles.

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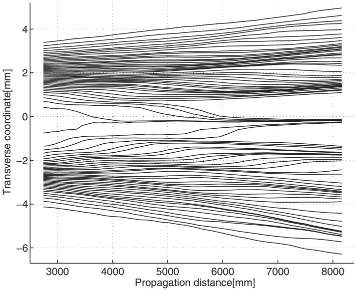


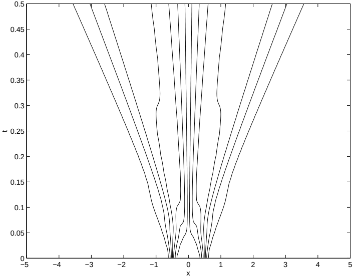


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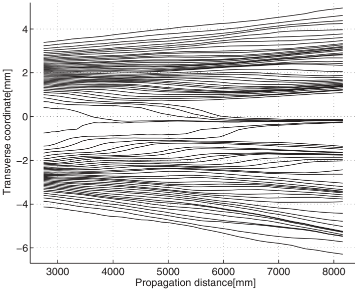


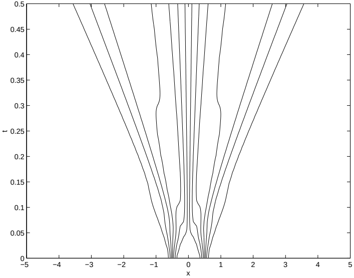


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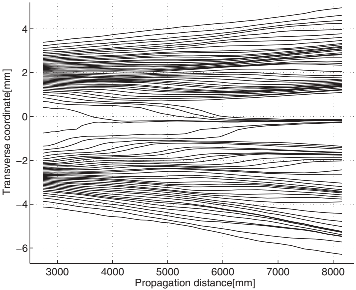


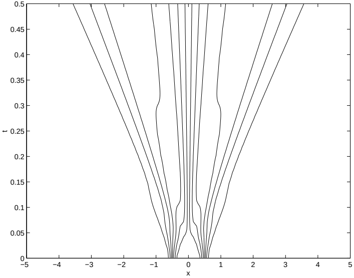


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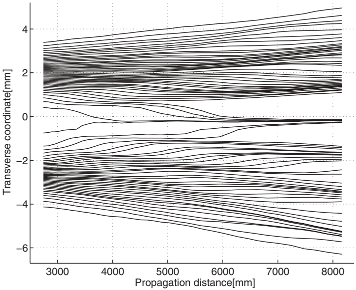


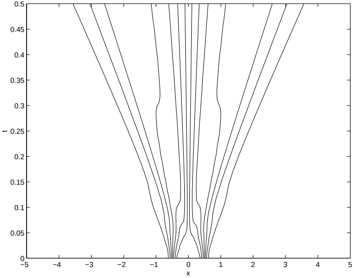


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# Outline

# A single particle; many worlds

- Consider a "world" comprising a *single* nonrelativistic *particle* of mass $m$, in one spatial dimension with potential $V(q)$.
- Let there be $N \gg 1$ worlds $\{x^n\}_{n=1}^N$: $x_n < x_{n+1}$, for all $n$.
- Say the $x^n(t_0)$ are arranged with a *slowly varying* inter-world separation, $s_0^n \equiv x^{n+1}(t_0) - x^n(t_0) \sim N^{-1}$:

$$s_0^n - s_0^{n+1} = s_0^n \times O(N^{-1})$$

- Say the initial velocities $v_0^n = \dot{x}^n(t_0)$ are also smoothly varying:

$$v_0^n - v_0^{n+1} = v_0^n \times O(N^{-1}).$$

- The initial density and velocities of these worlds correspond to a virtual ensemble of $N$ Bohmian particles for the wavefunction

$$\psi_0(x_n) = [N s_0^n]^{-1/2} \exp\left[ i \frac{m}{\hbar} \sum_{n'=1}^{n-1} v_0^{n'} s_0^{n'} \right].$$

# A single particle; many *interacting* worlds

- However,
  - *there is no wavefunction* in the ontology of our theory.
  - our ensemble of worlds is *real*, not virtual.
  - this is necessary because our worlds *interact*.
- The world-positions evolve via Newton's equations

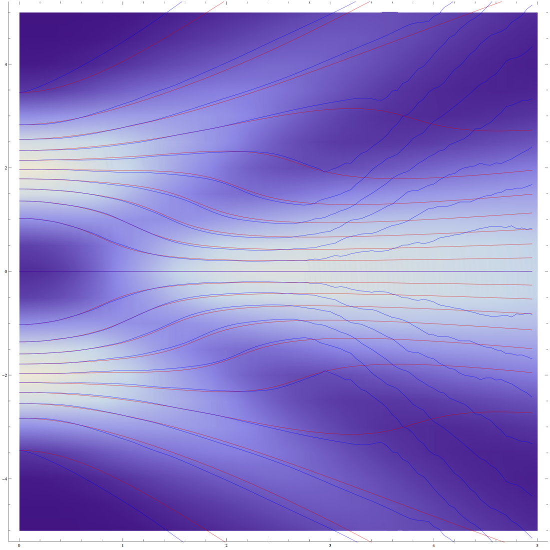
$$m\ddot{x}^n(t) = -\frac{\partial}{\partial x_n}\left[V(x^n) + \sum_{n'} Q_3\left(x^{n'-1}, x^{n'}, x^{n'+1}\right)\right].$$

- Here the "3-body" (3-world) "local" potential can be chosen as

$$Q_3\left(x^{n-1}, x^n, x^{n+1}\right) = \frac{\hbar^2}{8m}\left[\frac{1}{x_{n+1}-x_n} - \frac{1}{x_n - x_{n-1}}\right]^2.$$

- Then, in the limit $N \to \infty$, we believe we should recover the (virtual) Bohmian ensemble.

# It seems to work! up to some $t_{\mathrm{break}}(N)$.



Blue curves = sub-set of our many (100) interacting worlds, with $v_0^n \equiv 0$ and a bimodal distribution.

Contour shading of $|\psi_t|^2$ with $\psi_0$ as per $\{x_0^n, v_0^n\}$, evolved via Schrödinger's equation.

Red curves = corresponding Bohmian ensemble with velocities determined by

$$v_{\mathrm{Bohm}}(x) = \frac{\hbar}{m} \mathrm{Im} \frac{\psi_t'(x)}{\psi_t(x)}$$

# Analytical results from $E = \sum [m\dot{x}^2 + V + Q_3]$

- Ehrenfest's theorem, as in CM and QM, for all $N$,

$$\frac{d}{dt}\langle x\rangle = \frac{1}{m}\langle m\dot{x}\rangle, \qquad \frac{d}{dt}\langle m\dot{x}\rangle = -\langle V'(x)\rangle$$

  for the (real!) ensemble averages e.g. $\langle x\rangle \equiv N^{-1}\sum_{n=1}^{N} x^n$.
- Ensemble spreading

$$V_t[x] = V_0[x] + \frac{2t}{m}\mathrm{Cov}_0[x, m\dot{x}] + \frac{t^2}{m}\left[2\langle E\rangle - m\langle\dot{x}\rangle^2\right]$$

  as in QM and CM, for all $N$.
- Qualitative explanation for nonclassical barrier transmission *and* nonclassical reflection, via the quantum repulsion for $N > 1$.
- The harmonic oscillator ground configuration has an energy

$$\langle E\rangle = \frac{N-1}{N}\frac{\hbar\omega}{2},$$

  as in CM for $N = 1$ and as in QM in the limit $N \to \infty$.

# Outline

# Bohm's Bohmian Mechanics

- For simplicity consider a world comprising $P$ scalar nonrelativistic distinguishable particles, and no fields, in $D$-dimensional space.
- e.g. for $D = 3$ the $p$th particle has position $(q_{3p-2}, q_{3p-1}, q_{3p})^\top$.
- Define the world-configuration $\mathbf{q} = \{q_1, \cdots, q_K\}^\top$, $K = DP$.
- $i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{q}, t) = \left[\sum_{k=1}^{K} m_k \left(\frac{\partial}{\partial q_k}\right)^2 + V(\mathbf{q})\right]\Psi(\mathbf{q}, t)$.
- Then the *initial* world-configuration $\mathbf{x}(t_0)$ chosen at random from $|\Psi(\mathbf{q}, t_0)|^2$, and velocity $\dot{x}_k(t_0) = m^{-1}\mathrm{Re}\left[-i\hbar\frac{\partial}{\partial q_k}\ln\Psi(\mathbf{q}, t_0)\right]_{\mathbf{q}=\mathbf{x}(0)}$.
- The Bohmian world-configuration evolves according to

$$m_k\ddot{x}_k(t) = -\frac{\partial}{\partial q_k}[V(\mathbf{q}) + Q(\mathbf{q})]\Big|_{\mathbf{q}=\mathbf{x}(t)}$$

where $Q(\mathbf{q}) = |\Psi(\mathbf{q}, t_0)|^{-1}\sum_{k=1}^{K}\frac{-\hbar^2}{2m_k}\left(\frac{\partial}{\partial q_k}\right)^2|\Psi(\mathbf{q}, t_0)|$

# Many Bohmian Worlds

- Consider $N$ Bohmian worlds $\{\mathbf{x}^n\}_{n=1}^{N}$ each as above.
- $\frac{\partial}{\partial t}\Psi(\mathbf{q},t) = -i\left[\sum_{k=1}^{K} m_k \left(\frac{\partial}{\partial q_k}\right)^2 + V(\mathbf{q})\right]\Psi(\mathbf{q},t)$.
- Each initial $\mathbf{x}^n(t_0)$ chosen according to $|\Psi(\mathbf{x},t_0)|^2$, with velocity $\dot{x}_k^n(t_0) = \mathrm{Re}\left[-i\hbar\frac{\partial}{\partial q_k}\ln\Psi(\mathbf{q},t_0)\right]_{\mathbf{q}=\mathbf{x}^n(0)}$.
- Each world-particle evolves according to

$$m_k\ddot{x}_k^n(t) = -\frac{\partial}{\partial q_k}[V(\mathbf{q}) + Q(\mathbf{q})]\Big|_{\mathbf{q}=\mathbf{x}^n(t)}$$

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-
$$\lim_{N\to\infty} N^{-1}\sum_{n=1}^{N}\delta\left(\mathbf{q}-\mathbf{x}^n(t)\right) = |\Psi(\mathbf{q},t)|^2.$$

# Many *"Locally"* Interacting Worlds (No Wavefunction!)

- Consider $N$ worlds $\{\mathbf{x}^n\}_{n=1}^N$ with $N$ a *very* large integer.
- The set $\{\mathbf{x}^n(t_0)\}$ corresponds to *some* smooth distribution $\rho(\mathbf{x}, t_0)$.
- Each $\dot{x}_k^n(t_0) = \mathrm{Re}\left[-i\hbar \frac{\partial}{\partial q_k} \ln \Phi(\mathbf{q})\right]_{\mathbf{q}=\mathbf{x}^n(0)}$ for *some* smooth function $\Phi(\mathbf{q})$ satisfying $|\Phi(\mathbf{q})|^2 = \rho(\mathbf{q}, t_0)$.
- Each world-configuration evolves according to

$$m_k \ddot{x}_k^n(t) = -\left.\frac{\partial V(\mathbf{q})}{\partial q_k} + F_k^Q(\mathbf{q})\right|_{\mathbf{q}=\mathbf{x}^n(t)}$$

where $F_k^Q(\mathbf{q})$ is the $k$th component of an approximation to the quantum force determined by the *local* density of worlds (and its derivatives) in the vicinity of $\mathbf{q}$.

# Ontology and Epistemology

- All worlds are equally real.
- Your consciousness supervenes on only one of the worlds.
- Just as (here and in classical physics) your consciousness supervenes only on one part (i.e. you) of a world.
- There is no wavefunction and hence no collapse of the wavefunction.
- Effective wavefunction collapse is just Bayesian updating by some consciousness about *which world* it is likely to supervene upon.
- Agreement with standard QM emerges much the same as in BM or the (noninteracting) MWI.
- *All* quantum effects are a consequence of interaction between worlds so they *are* observable!

# Outline

# Summary

- Weak values shed new light on fundamental questions in QM.
- In particular they allow one to *empirically* obtain a unique Bohmian velocity law, and thereby also single out the configuration as the unique Bohmian reality.
- The (empirically determinable) ensemble of Bohmian trajectories suggests the evolution of a physical ensemble of particles with a repulsive interaction.
- This intuition can be made precise, at least in the 1D case, with an explicit 3-body interaction.
- We believe this can be generalized so that QM *emerges* from Many Interacting Worlds.

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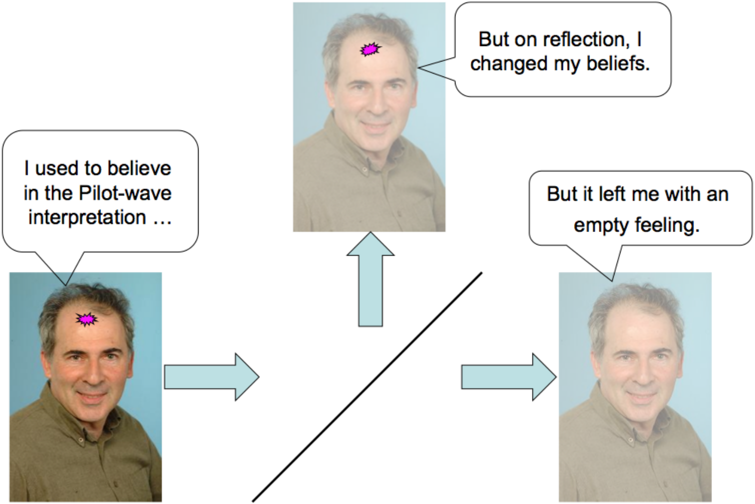
# Why go past Bohmian Mechanics?

- The wavefunction has the *awkward* role of both guiding the evolution of the world-configuration and specifying a probability or typicality measure for it.
- It is *not clear* what a probability measure means for a single world.
- *Unlike* Bohm's hope, it has not led to a post-quantum theory with definite new predictions.
- It has been criticized as just "tacking on" something to the usual formalism, rather than having quantum mechanics *emerge* from something quite different.
- The wavefunction exists in all of these places where the world-configuration isn't. i.e. the theory has ...

# ... empty waves.




<< One of us (L.V.) used to view the Bohm interpretation as the most elegant way of pointing out one of the many worlds of the Everett interpretation as "real" >> --- Aharonov and Vaidman (1996).

# Expected and Unexpected(?) Property of WVs.

- Expected property: **linearity** —

$$\hat{C} = \hat{A} + \hat{B} \implies {}_{E^{\rm f}}\langle C^{\rm w}\rangle_{\rho^{\rm in}} = {}_{E^{\rm f}}\langle A^{\rm w}\rangle_{\rho^{\rm in}} + {}_{E^{\rm f}}\langle B^{\rm w}\rangle_{\rho^{\rm in}} .$$

- Expected property: **consistency** with strong measurements — if, with pre- ($\rho^{\rm in}$) and post- ($\hat{E}^{\rm f}$) selection a strong measurement of $A$ *always* would yield the answer $\lambda$, then ${}_{E^{\rm f}}\langle A^{\rm w}\rangle_{\rho^{\rm in}} = \lambda$.

- Unexpected(?) property: **anomalous weak values** — it is **not** a theorem that

$$\lambda_{\rm min}(\hat{A}) \le {}_{E^{\rm f}}\langle A^{\rm w}\rangle_{\rho^{\rm in}} \le \lambda_{\rm max}(\hat{A}).$$

Hence

**How the Result of a Measurement of a Component of the Spin of a Spin- $\frac{1}{2}$ Particle Can Turn Out to be 100**

Yakir Aharonov, David Z. Albert, and Lev Vaidman

# Just the trajectories